

## Core 1

### Indices

1)

State the value of each of the following.

(i)  $2^{-3}$  [1]

(ii)  $9^0$  [1]

2)

(i) Simplify  $(5a^2b)^3 \times 2b^4$ . [2]

(ii) Evaluate  $(\frac{1}{16})^{-1}$ . [1]

3)

Simplify  $\frac{(3xy^4)^3}{6x^5y^2}$ . [3]

4)

(i) Write down the value of  $(\frac{1}{4})^0$ . [1]

(ii) Find the value of  $16^{-\frac{3}{2}}$ . [3]

5)

Find the value of  $(\frac{1}{2})^{-5}$ . [2]

6)

Find the value of  $(\frac{1}{25})^{-\frac{1}{2}}$ . [2]

7)

Find the value of each of the following, giving each answer as an integer or fraction as appropriate.

(i)  $25^{\frac{3}{2}}$  [2]

(ii)  $(\frac{7}{3})^{-2}$  [2]

8)

(i) Evaluate  $(\frac{9}{16})^{-\frac{1}{2}}$ . [2]

(ii) Simplify  $\frac{(2ac^2)^3 \times 9a^2c}{36a^4c^{12}}$ . [3]

9)

(i) Express  $125\sqrt{5}$  in the form  $5^k$ . [2]

(ii) Simplify  $(4a^3b^5)^2$ . [2]

## Surds

1)

(i) Simplify  $\frac{\sqrt{48}}{2\sqrt{27}}$ . [2]

(ii) Expand and simplify  $(5 - 3\sqrt{2})^2$ . [3]

2)

(i) Express  $\sqrt{75} + \sqrt{48}$  in the form  $a\sqrt{3}$ . [2]

(ii) Express  $\frac{14}{3 - \sqrt{2}}$  in the form  $b + c\sqrt{d}$ . [3]

3)

(i) Simplify  $\sqrt{98} - \sqrt{50}$ . [2]

(ii) Express  $\frac{6\sqrt{5}}{2 + \sqrt{5}}$  in the form  $a + b\sqrt{5}$ , where  $a$  and  $b$  are integers. [3]

4)

(i) Express  $\sqrt{48} + \sqrt{27}$  in the form  $a\sqrt{3}$ . [2]

(ii) Simplify  $\frac{5\sqrt{2}}{3 - \sqrt{2}}$ . Give your answer in the form  $\frac{b + c\sqrt{2}}{d}$ . [3]

5)

(i) Express  $\frac{1}{5 + \sqrt{3}}$  in the form  $\frac{a + b\sqrt{3}}{c}$ , where  $a$ ,  $b$  and  $c$  are integers. [2]

(ii) Expand and simplify  $(3 - 2\sqrt{7})^2$ . [3]

6)

You are given that  $a = \frac{3}{2}$ ,  $b = \frac{9 - \sqrt{17}}{4}$  and  $c = \frac{9 + \sqrt{17}}{4}$ . Show that  $a + b + c = abc$ . [4]

## Algebraic fractions

1)

Factorise  $x^2 - 4$  and  $x^2 - 5x + 6$ .

Hence express  $\frac{x^2 - 4}{x^2 - 5x + 6}$  as a fraction in its simplest form. [3]

2)

Factorise and hence simplify  $\frac{3x^2 - 7x + 4}{x^2 - 1}$ . [3]

## Proof

1)

$n$  is a positive integer. Show that  $n^2 + n$  is always even. [2]

2)

Prove that, when  $n$  is an integer,  $n^3 - n$  is always even. [3]

3)

(i) Prove that 12 is a factor of  $3n^2 + 6n$  for all even positive integers  $n$ . [3]

(ii) Determine whether 12 is a factor of  $3n^2 + 6n$  for all positive integers  $n$ . [2]

4)

Factorise  $n^3 + 3n^2 + 2n$ . Hence prove that, when  $n$  is a positive integer,  $n^3 + 3n^2 + 2n$  is always divisible by 6. [3]

### Solving linear inequalities

- 1)  
Solve the inequality  $6(x + 3) > 2x + 5$ . [3]
- 2)  
Solve the inequality  $3x - 1 > 5 - x$ . [2]
- 3)  
Solve the inequality  $\frac{5x - 3}{2} < x + 5$ . [3]
- 4)  
Solve the inequality  $\frac{3(2x + 1)}{4} > -6$ . [4]
- 5)  
Solve the inequality  $7 - x < 5x - 2$ . [3]
- 6)  
Solve the inequality  $1 - 2x < 4 + 3x$ . [3]

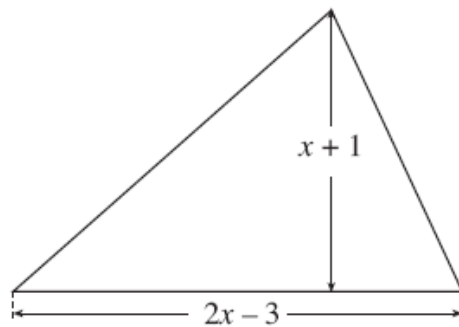
### Solving equations

- 1)  
Solve the equation  $\frac{4x + 5}{2x} = -3$ . [3]
- 2)  
Solve the equation  $\frac{3x + 1}{2x} = 4$ . [3]
- 3)  
Solve the equation  $y^2 - 7y + 12 = 0$ .  
Hence solve the equation  $x^4 - 7x^2 + 12 = 0$ . [4]
- 4)  
Solve the equation  $4x^2 + 20x + 25 = 0$ . [2]
- 5)  
Solve the equation  $2x^2 + 3x = 0$ . [2]

Forming and solving equations

1)

The triangle shown in Fig. 10 has height  $(x + 1)$  cm and base  $(2x - 3)$  cm. Its area is  $9 \text{ cm}^2$ .



Not to scale

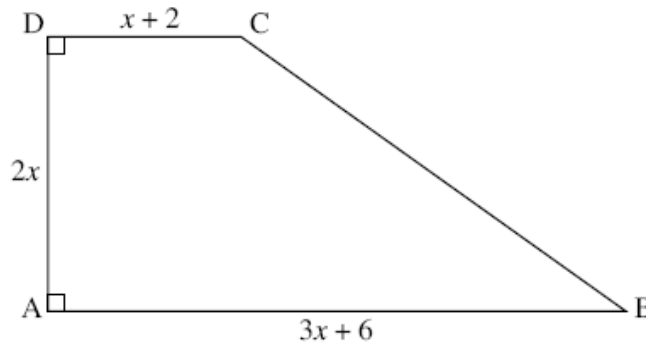
**Fig. 10**

(i) Show that  $2x^2 - x - 21 = 0$ . [2]

(ii) By factorising, solve the equation  $2x^2 - x - 21 = 0$ . Hence find the height and base of the triangle. [3]

2)

Fig. 9 shows a trapezium ABCD, with the lengths in centimetres of three of its sides.



**Fig. 9**

This trapezium has area  $140 \text{ cm}^2$ .

(i) Show that  $x^2 + 2x - 35 = 0$ . [2]

(ii) Hence find the length of side AB of the trapezium. [3]

### Completing the square and turning points

1)

(i) Express  $x^2 + 6x + 5$  in the form  $(x + a)^2 + b$ . [3]

(ii) Write down the coordinates of the minimum point on the graph of  $y = x^2 + 6x + 5$ . [2]

2)

(i) Write  $x^2 - 7x + 6$  in the form  $(x - a)^2 + b$ . [3]

(ii) State the coordinates of the minimum point on the graph of  $y = x^2 - 7x + 6$ . [2]

(iii) Find the coordinates of the points where the graph of  $y = x^2 - 7x + 6$  crosses the axes and sketch the graph. [5]

3)

(i) Write  $3x^2 + 6x + 10$  in the form  $a(x + b)^2 + c$ . [4]

(ii) Hence or otherwise, show that the graph of  $y = 3x^2 + 6x + 10$  is always above the  $x$ -axis. [2]

4)

(i) Write  $4x^2 - 24x + 27$  in the form  $a(x - b)^2 + c$ . [4]

(ii) State the coordinates of the minimum point on the curve  $y = 4x^2 - 24x + 27$ . [2]

(iii) Solve the equation  $4x^2 - 24x + 27 = 0$ . [3]

(iv) Sketch the graph of the curve  $y = 4x^2 - 24x + 27$ . [3]

### Discriminant and roots

1)

Find the discriminant of  $3x^2 + 5x + 2$ . Hence state the number of distinct real roots of the equation  $3x^2 + 5x + 2 = 0$ . [3]

2)

Find the set of values of  $k$  for which the equation  $2x^2 + 3x - k = 0$  has no real roots. [3]

3)

Find the set of values of  $k$  for which the equation  $2x^2 + kx + 2 = 0$  has no real roots. [4]

4)

Prove that the line  $y = 3x - 10$  does not intersect the curve  $y = x^2 - 5x + 7$ . [5]

## Changing the subject of a formula

1)

Make  $a$  the subject of the formula  $s = ut + \frac{1}{2}at^2$ . [3]

2)

The volume of a cone is given by the formula  $V = \frac{1}{3}\pi r^2 h$ . Make  $r$  the subject of this formula. [3]

3)

Make  $y$  the subject of the formula  $a = \frac{\sqrt{y} - 5}{c}$ . [3]

4)

Rearrange the formula  $c = \sqrt{\frac{a+b}{2}}$  to make  $a$  the subject. [3]

5)

The volume  $V$  of a cone with base radius  $r$  and slant height  $l$  is given by the formula

$$V = \frac{1}{3}\pi r^2 \sqrt{l^2 - r^2}.$$

Rearrange this formula to make  $l$  the subject. [4]

6)

Make  $a$  the subject of the equation

$$2a + 5c = af + 7c. [3]$$

7)

Rearrange  $y + 5 = x(y + 2)$  to make  $y$  the subject of the formula. [4]

8)

Make  $C$  the subject of the formula  $P = \frac{C}{C+4}$ . [4]

9)

Make  $x$  the subject of the formula  $y = \frac{1-2x}{x+3}$ . [4]

## Equation of a straight line

1)

Find the equation of the line which is parallel to  $y = 5x - 4$  and which passes through the point  $(2, 13)$ .  
Give your answer in the form  $y = ax + b$ . [3]

2)

Find the equation of the line which is parallel to  $y = 3x + 1$  and which passes through the point with coordinates  $(4, 5)$ . [3]

3)

Find the equation of the line passing through  $(-1, -9)$  and  $(3, 11)$ . Give your answer in the form  $y = mx + c$ . [3]

4)

A line has equation  $3x + 2y = 6$ . Find the equation of the line parallel to this which passes through the point  $(2, 10)$ . [3]

5)

(i) Find the equation of the line passing through A  $(-1, 1)$  and B  $(3, 9)$ . [3]

(ii) Show that the equation of the perpendicular bisector of AB is  $2y + x = 11$ . [4]

## Intersection of two lines

1)

Find the coordinates of the point of intersection of the lines  $y = 3x + 1$  and  $x + 3y = 6$ . [3]

2)

Find, algebraically, the coordinates of the point of intersection of the lines  $y = 2x - 5$  and  $6x + 2y = 7$ . [4]

3)

Solve the simultaneous equations  $y = x^2 - 6x + 2$  and  $y = 2x - 14$ . Hence show that the line  $y = 2x - 14$  is a tangent to the curve  $y = x^2 - 6x + 2$ . [5]

4)

Find algebraically the coordinates of the points of intersection of the curve  $y = 4x^2 + 24x + 31$  and the line  $x + y = 10$ . [5]

5)

Find the coordinates of the points of intersection of the circle  $x^2 + y^2 = 25$  and the line  $y = 3x$ .  
Give your answers in surd form. [5]

6)

A circle has equation  $x^2 + y^2 = 45$ .

(i) State the centre and radius of this circle. [2]

(ii) The circle intersects the line with equation  $x + y = 3$  at two points, A and B. Find algebraically the coordinates of A and B.

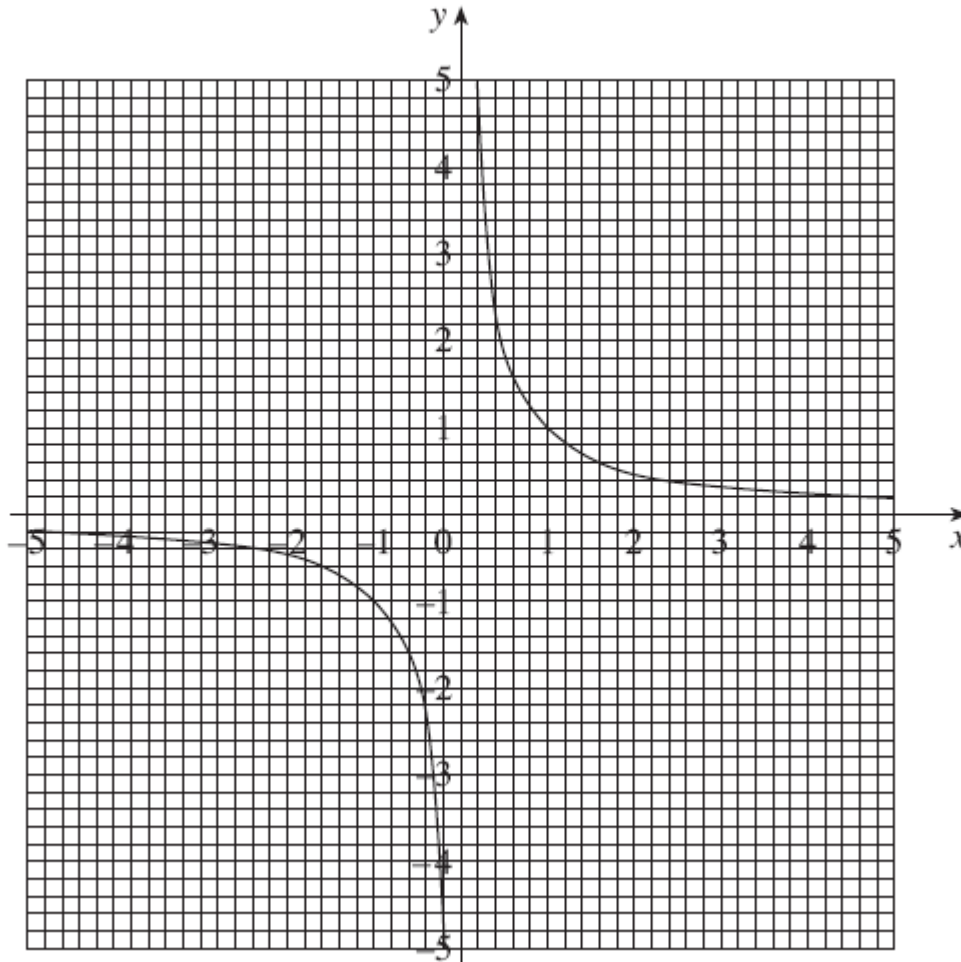
Show that the distance AB is  $\sqrt{162}$ . [8]



Using graphs to solve equations

1)

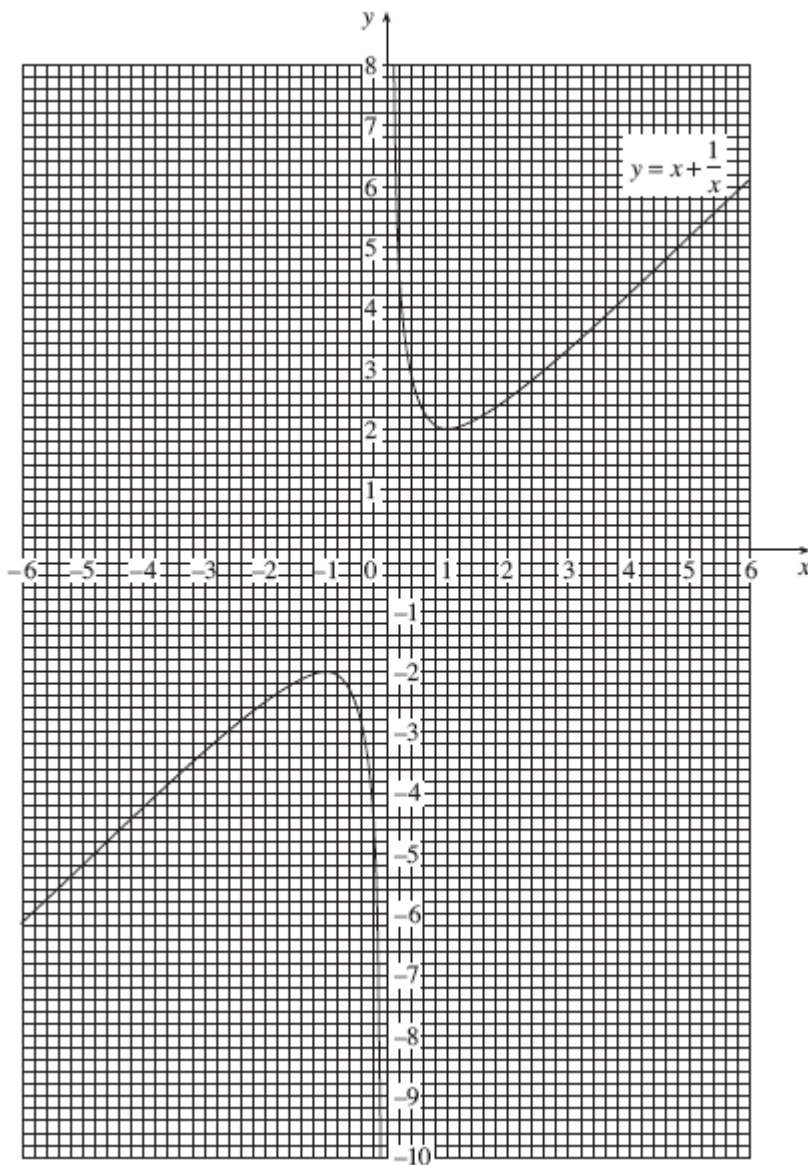
The insert shows the graph of  $y = \frac{1}{x}$ ,  $x \neq 0$ .



- (i) Use the graph to find approximate roots of the equation  $\frac{1}{x} = 2x + 3$ , showing your method clearly. [3]
- (ii) Rearrange the equation  $\frac{1}{x} = 2x + 3$  to form a quadratic equation. Solve the resulting equation, leaving your answers in the form  $\frac{p \pm \sqrt{q}}{r}$ . [5]
- (iii) Draw the graph of  $y = \frac{1}{x} + 2$ ,  $x \neq 0$ , on the grid used for part (i). [2]

2)

The graph of  $y = x + \frac{1}{x}$  is shown on the insert. The lowest point on one branch is  $(1, 2)$ . The highest point on the other branch is  $(-1, -2)$ .



Use the graph to solve the following equations, showing your method clearly.

(A)  $x + \frac{1}{x} = 4$  [2]

(B)  $2x + \frac{1}{x} = 4$  [4]

## Core 1 and Core 2

### Transformation of graphs

1)

The point P (5, 4) is on the curve  $y = f(x)$ . State the coordinates of the image of P when the graph of  $y = f(x)$  is transformed to the graph of

(i)  $y = f(x - 5)$ , [2]

(ii)  $y = f(x) + 7$ . [2]

2)

The curve  $y = f(x)$  has a minimum point at (3, 5).

State the coordinates of the corresponding minimum point on the graph of

(i)  $y = 3f(x)$ , [2]

(ii)  $y = f(2x)$ . [2]

3)

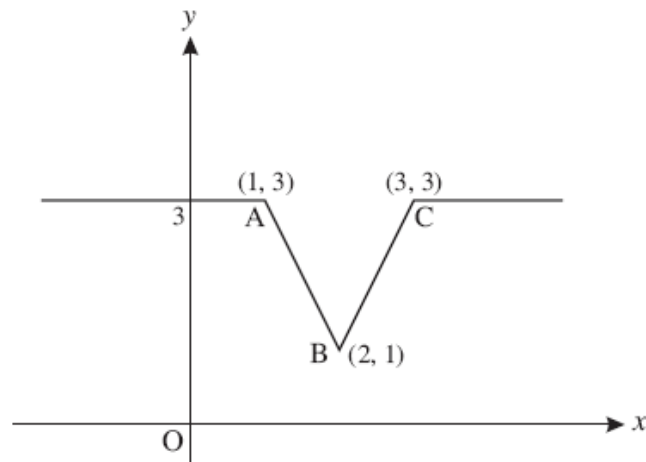


Fig. 4

Fig. 4 shows a sketch of the graph of  $y = f(x)$ . On separate diagrams, sketch the graphs of the following, showing clearly the coordinates of the points corresponding to A, B and C.

(i)  $y = 2f(x)$  [2]

(ii)  $y = f(x + 3)$  [2]

4)

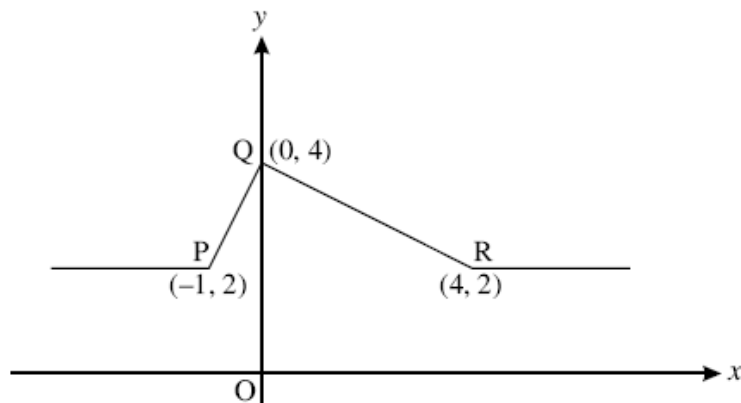


Fig. 5

Fig. 5 shows a sketch of the graph of  $y = f(x)$ . On separate diagrams, sketch the graphs of the following, showing clearly the coordinates of the points corresponding to P, Q and R.

(i)  $y = f(2x)$  [2]

(ii)  $y = \frac{1}{4}f(x)$  [2]

5)

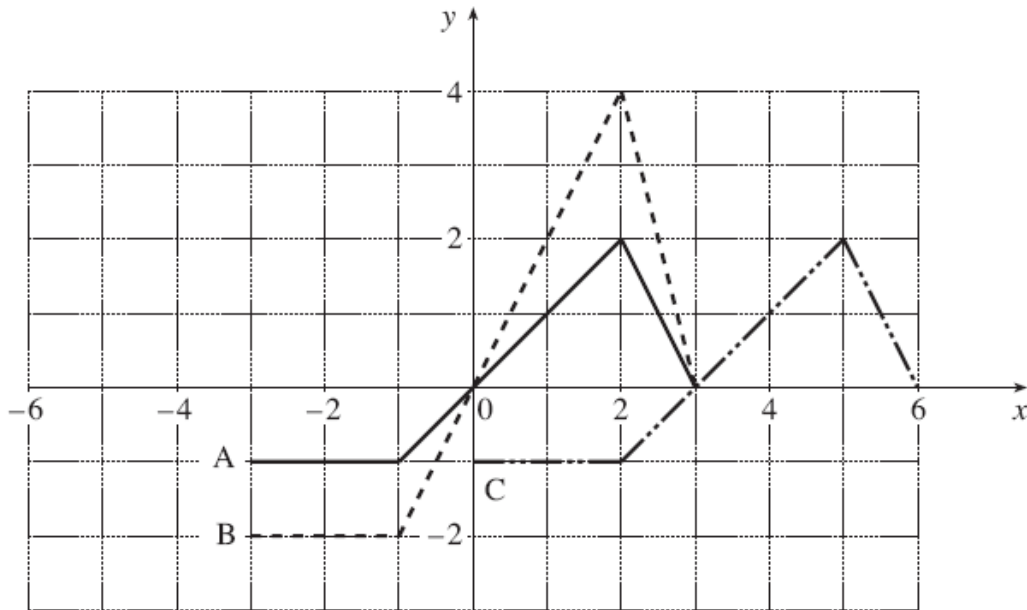


Fig. 3

Fig. 3 shows sketches of three graphs, A, B and C. The equation of graph A is  $y = f(x)$ .

State the equation of

(i) graph B, [2]

(ii) graph C. [2]

6)

The curve  $y = x^2 - 4$  is translated by  $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ .

Write down an equation for the translated curve. You need not simplify your answer. [2]

7)

Describe fully the transformation which maps the curve  $y = x^2$  onto the curve  $y = (x + 4)^2$ . [2]

## Core 2

### Trigonometric graphs

1)

Sketch the curve  $y = \sin x$  for  $0^\circ \leq x \leq 360^\circ$ .

Solve the equation  $\sin x = -0.68$  for  $0^\circ \leq x \leq 360^\circ$ . [4]

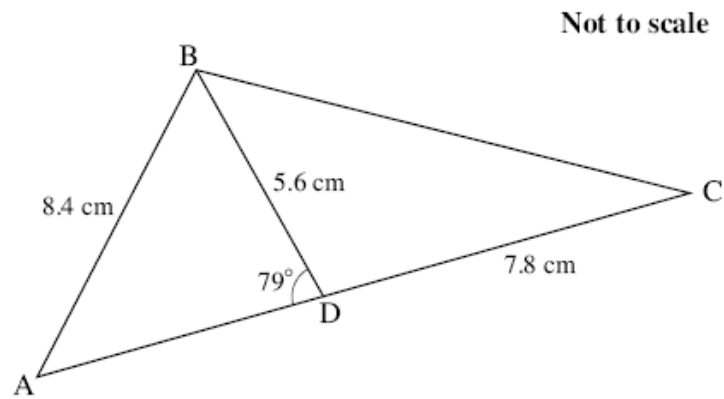
2)

Sketch the graph of  $y = \cos x$  for  $0^\circ \leq x \leq 360^\circ$ .

On the same axes, sketch the graph of  $y = \cos 2x$  for  $0^\circ \leq x \leq 360^\circ$ . Label each graph clearly. [3]

Sine rule and cosine rule

1)



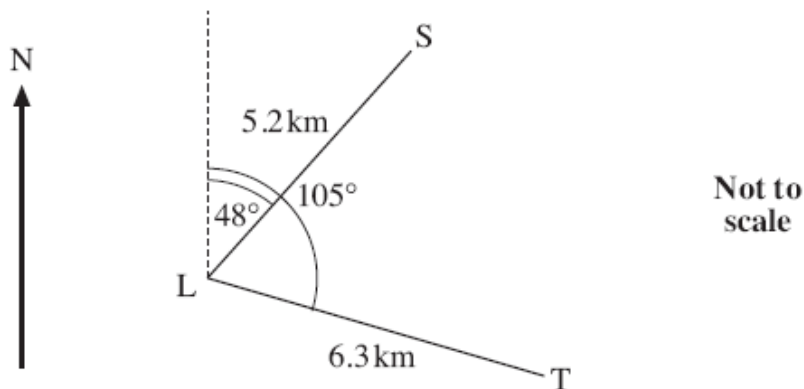
**Fig. 7**

Fig. 7 shows triangle ABC, with  $AB = 8.4$  cm. D is a point on AC such that angle  $ADB = 79^\circ$ ,  $BD = 5.6$  cm and  $CD = 7.8$  cm.

Calculate

- (i) angle BAD, [2]
- (ii) the length BC. [3]

2)



**Fig. 10.1**

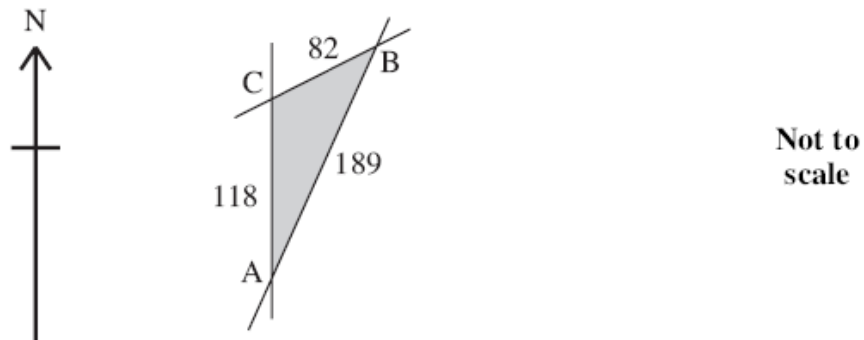
At a certain time, ship S is 5.2 km from lighthouse L on a bearing of  $048^\circ$ . At the same time, ship T is 6.3 km from L on a bearing of  $105^\circ$ , as shown in Fig. 10.1.

For these positions, calculate

- (A) the distance between ships S and T, [3]
- (B) the bearing of S from T. [3]

3)

Fig. 11.1 shows a village green which is bordered by 3 straight roads AB, BC and CA. The road AC runs due North and the measurements shown are in metres.



**Fig. 11.1**

(i) Calculate the bearing of B from C, giving your answer to the nearest  $0.1^\circ$ . [4]

(ii) Calculate the area of the village green. [2]

### Arc length and sector area

1)

A sector of a circle of radius 18.0 cm has arc length 43.2 cm.

Find the angle of the sector. [2]

2)

A sector of a circle of radius 5 cm has area  $9 \text{ cm}^2$ .

Find the perimeter of the sector. [5]