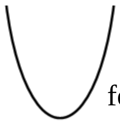
# Sketching quadratic graphs

**A LEVEL LINKS**
**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.

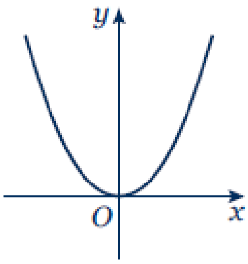

for $a > 0$

for $a < 0$

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the $y$-axis substitute $x = 0$ into the function.
- To find where the curve intersects the $x$-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

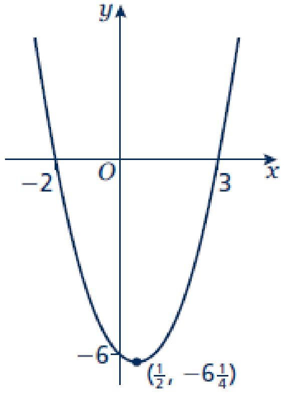
## Examples

**Example 1** Sketch the graph of $y = x^2$.

| | |
|---|---|
|  | The graph of $y = x^2$ is a parabola.<br><br>When $x = 0$, $y = 0$.<br><br>$a = 1$ which is greater than zero, so the graph has the shape:  |

**Example 2** Sketch the graph of $y = x^2 - x - 6$.

| | |
|---|---|
| When $x = 0$, $y = 0^2 - 0 - 6 = -6$<br>So the graph intersects the $y$-axis at $(0, -6)$<br>When $y = 0$, $x^2 - x - 6 = 0$ | **1** Find where the graph intersects the $y$-axis by substituting $x = 0$.<br><br>**2** Find where the graph intersects the $x$-axis by substituting $y = 0$. |
| $(x + 2)(x - 3) = 0$ | **3** Solve the equation by factorising. |
| $x = -2$ or $x = 3$ | **4** Solve $(x + 2) = 0$ and $(x - 3) = 0$. |
| So,<br>the graph intersects the $x$-axis at $(-2, 0)$ and $(3, 0)$ | **5** $a = 1$ which is greater than zero, so the graph has the shape:  |
| | *(continued on next page)* |

| | |
|---|---|
| $x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ <br> $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$ <br> When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and <br> $y = -\frac{25}{4}$, so the turning point is at the <br> point $\left(\frac{1}{2}, -\frac{25}{4}\right)$ <br>   | **6** To find the turning point, complete the square. <br> **7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero. |

## Practice

**1** Sketch the graph of $y = -x^2$.

**2** Sketch each graph, labelling where the curve crosses the axes.

**a** $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

**3** Sketch each graph, labelling where the curve crosses the axes.

**a** $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

**d** $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

**4** Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

## Extend

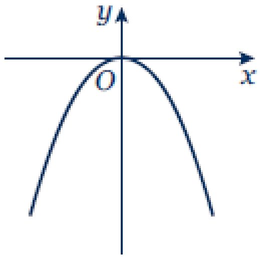
**5** Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

**a** $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

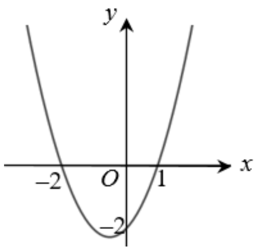
**6** Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

## Answers

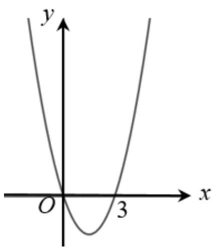
**1**



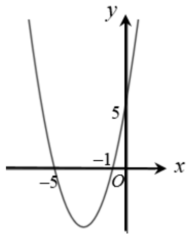

**2** **a**



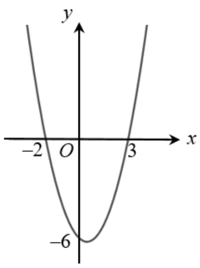

**b**



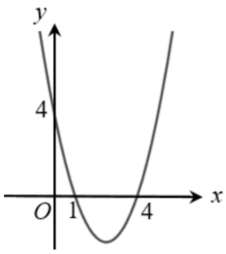

**c**



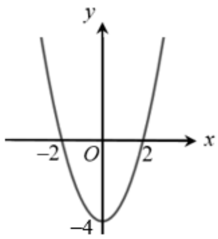

**3** **a**



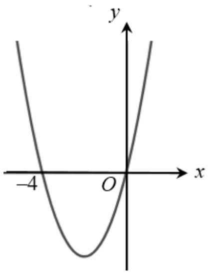

**b**



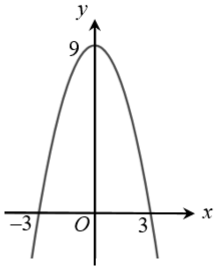

**c**



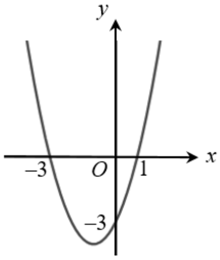

**d**



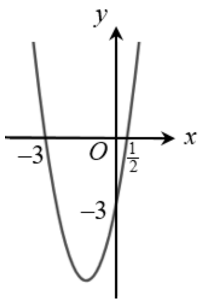

**e**



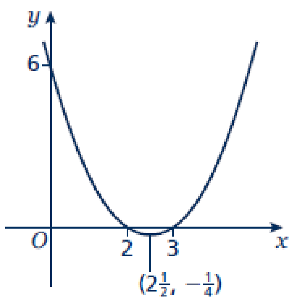

**f**



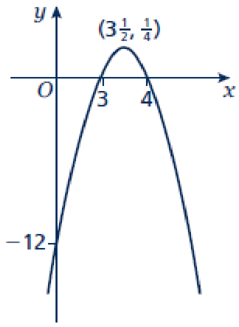
**4**



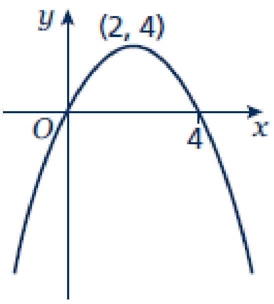

**5** **a**



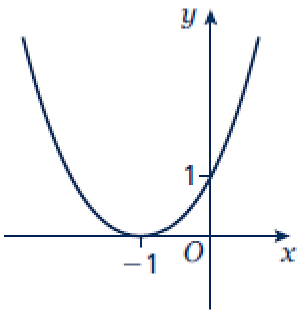

**b**




**c**




**6**




Line of symmetry at $x = -1$.