AS Topics

Constant Acceleration (SUVAT)	Variable Acceleration
Forces	Connected Particles
Distance/Speed Time Graphs	

Contains:
AS SAMs
AS 2018
AS 2019
AS 2020
AS 2021
AS 2022
AS 2023
AS 2024

A2 Topics

Projectiles	Variable Acceleration (including A2 functions and vectors)
Forces (including slopes and friction)	Connected Particles (including slopes and friction)
Moments	Constant Acceleration with Vectors

Contains:
A2 SAMs
A2 2018
A2 2019
A2 2020
A2 2021
A2 2022
A2 2023
A2 2024

Constant Acceleration (SUVAT)

7. A car is moving along a straight horizontal road with constant acceleration.

There are three points A, B and C, in that order, on the road, where $AB = 22$ m and $BC = 104$ m.

The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(7)

Question	Scheme	Marks	AOs
7(i)(ii)	Using a correct strategy for solving the problem by setting up two equations in a and u only and solving for either	M1	3.1b
	Equation in a and u only	M1	3.1b
	$22 = 2u + \frac{1}{2} a 2^2$	A1	1.1b
	Another equation in a and u only	M1	3.1b
	$126 = 6u + \frac{1}{2} a 6^2$	A1	1.1b
	5 m s^{-2}	A1	1.1b
	6 m s^{-1}	A1 ft	1.1b
			(7 marks)

6. A man throws a tennis ball into the air so that, at the instant when the ball leaves his hand, the ball is 2 m above the ground and is moving vertically upwards with speed $9\,\mathrm{m\,s^{-1}}$

The motion of the ball is modelled as that of a particle moving freely under gravity and the acceleration due to gravity is modelled as being of constant magnitude $10\,\mathrm{m\,s^{-2}}$

The ball hits the ground T seconds after leaving the man's hand.

Using the model, find the value of T.

(4)

Question	Scheme	Marks	AOs
6.	Equation in *t* only	M1	2.1
	$-2 = 9t - \frac{1}{2} \times 10t^2$	A1	1.1b
	$5t^2 - 9t - 2 = 0 = (5t+1)(t-2)$	DM1	1.1b
	$T = 2$ (only)	A1	1.1b
		(4)	
		(4 marks)	

1. At time $t = 0$, a small ball is projected vertically upwards with speed $U\,\text{m s}^{-1}$ from a point A that is 16.8 m above horizontal ground.

The speed of the ball at the instant immediately before it hits the ground for the first time is $19\,\text{m s}^{-1}$

The ball hits the ground for the first time at time $t = T$ seconds.

The motion of the ball, from the instant it is projected until the instant just before it hits the ground for the first time, is modelled as that of a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$

Using the model,

(a) show that $U = 5$

(2)

(b) find the value of T,

(2)

(c) find the time from the instant the ball is projected until the instant when the ball is 1.2 m below A.

(4)

(d) Sketch a velocity-time graph for the motion of the ball for $0 \leqslant t \leqslant T$, stating the coordinates of the start point and the end point of your graph.

(2)

In a refinement of the model of the motion of the ball, the effect of air resistance on the ball is included and this refined model is now used to find the value of U.

(e) State, with a reason, how this new value of U would compare with the value found in part (a), using the initial unrefined model.

(1)

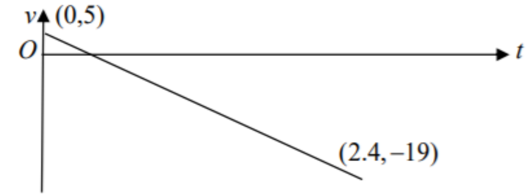
(f) Suggest one further refinement that could be made to the model, apart from including air resistance, that would make the model more realistic.

(1)

AS 2020

Constant Accn. SUVAT

1.(a)	$19^2 = (-U)^2 + 2\times10\times16.8$ (Allow use of $g = 9.8$ for this M mark)	M1
	$U = 5$ *	A1*
		(2)
	For consistent use of $g = 9.8$ in parts (b), (c) and (d), treat as a MR. i.e. max (b) M1A0 (c)M1A0M(A)0A1ft (d)B1B1ft	
(b)	$19 = -5 + 10T$ **OR** $16.8 = \frac{(-5+19)}{2}T$ **OR** $16.8 = -5T + \frac{1}{2}\times10T^2$ **OR** $16.8 = 19T - \frac{1}{2}\times10T^2$	M1
	$T = 2.4$	A1
		(2)
(c)	$1.2 = -5t + \frac{1}{2}\times10\times t^2$	M1
	$5t^2 - 5t - 1.2 = 0$	A1 M(A)1
	$t = 1.2$ (s)	A1
		(4)

(d)		B1 shape
	(0,5) and (2.4, −19) Allow these to be marked on the axes.	B1ft
		(2)
(e)	Greater since air resistance would slow the ball down.	B1
		(1)
(f)	Take into account: spin, wind effects, use a more accurate value of *g*, not model the ball as a particle	B1
		(1)

HOME

1. At time $t = 0$, a small stone is thrown vertically upwards with speed $14.7\,\text{m}\,\text{s}^{-1}$ from a point A.

At time $t = T$ seconds, the stone passes through A, moving downwards.

The stone is modelled as a particle moving freely under gravity throughout its motion.

Using the model,

(a) find the value of T,

(2)

(b) find the total distance travelled by the stone in the first 4 seconds of its motion.

(4)

(c) State one refinement that could be made to the model, apart from air resistance, that would make the model more realistic.

(1)

AS 2021

Constant Accn. SUVAT

Question	Scheme	Marks	AOs
1.(a)	$14.7 = -14.7 + 9.8T$ or $0 = 14.7T - \frac{1}{2} \times 9.8T^2$ or $0 = 14.7 - 9.8 \times \left(\frac{1}{2}T\right)$ oe	M1	3.4
	$T = 3$	A1	1.1b
		(2)	
(b)	$s_1 = \frac{(14.7+0)}{2} \times 1.5$ (11.025 or $\frac{441}{40}$)	M1	1.1b
	$s_2 = \frac{1}{2} \times 9.8 \times 2.5^2$ (30.625 or $\frac{245}{8}$) **OR** $s_3 = 14.7 \times 1 + \frac{1}{2} \times 9.8 \times 1^2$ (19.6 or $\frac{98}{5}$) **OR** $-s_3 = 14.7 \times 4 - \frac{1}{2} \times 9.8 \times 4^2$ (- 19.6) (allow omission of – on LHS)	M1	1.1b
	Total distance = $s_1 + s_2$ **OR** $2s_1 + s_3$	M1	2.1
	= 41.7 m or 42 m	A1	1.1b
		(4)	
(c)	e.g. Take account of the dimensions of the stone (e.g. allow for spin), do not model the stone as a particle, use a more accurate value for g	B1	3.5c
		(1)	
		(7 marks)	

HOME

1. The point A is 1.8 m vertically above horizontal ground.

At time $t = 0$, a small stone is projected vertically upwards with speed $U\,\text{m s}^{-1}$ from the point A.

At time $t = T$ seconds, the stone hits the ground.

The speed of the stone as it hits the ground is $10\,\text{m s}^{-1}$

In an initial model of the motion of the stone as it moves from A to where it hits the ground

- the stone is modelled as a particle moving freely under gravity
- **the acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$**

Using the model,

(a) find the value of U,

(3)

(b) find the value of T.

(2)

(c) Suggest one refinement, apart from including air resistance, that would make the model more realistic.

(1)

In reality the stone will not move freely under gravity and will be subject to air resistance.

(d) Explain how this would affect your answer to part (a).

(1)

AS 2022

Constant Accn. SUVAT

Question	Scheme	Marks	AOs
1(a)	Complete method to produce an equation in U only	M1	3.4
	e.g. $10^2 = U^2 + 2 \times g \times 1.8$ oe	A1	1.1b
	OR a **complete** method where they find T first and use it to find an equation in U only M1		
	A correct equation in U only. A1		
	$U = 8$ (<u>only</u> this answer)	A1	1.1b
		(3)	
(b)	Complete method to find an equation in T only: $10 = -8 + gT$ or $1.8 = 10T - \frac{1}{2}gT^2$ or $1.8 = \frac{(-8+10)}{2}T$ or $1.8 = -8T + \frac{1}{2}gT^2$	M1	3.4
	$T = 1.8$ oe e.g. 9/5	A1	1.1b
		(2)	
(c)	e.g. Use a more accurate (less rounded) value for g (or gravity), use $g = 9.8$ or $g = 9.81$, allow for wind effects, allow for the spin of the stone, include dimensions of stone (not a particle), shape and/or size of stone, allow for variable acceleration. If air resistance is mentioned as an extra, ignore it.	B1	3.5c
(d)	U would be greater. Allow without U, e.g it would be greater, or just 'greater' oe ISW	B1	3.5a
		(1)	
			(7 marks)

HOME

2. A small stone is projected vertically upwards with speed $39.2\,\text{m}\,\text{s}^{-1}$ from a point O.

The stone is modelled as a particle moving freely under gravity from when it is projected until it hits the ground 10 s later.

Using the model, find

(a) the height of O above the ground,

(3)

(b) the total length of time for which the speed of the stone is less than or equal to $24.5\,\text{m}\,\text{s}^{-1}$

(3)

(c) State one refinement that could be made to the model that would make your answer to part (a) more accurate.

(1)

HOME

If they use g = 9.81 or 10 in this question, penalise once for whole question.

Question	Scheme	Marks	AOs
2(a)	Attempt to find the displacement after 10 s	M1	3.1b
	$39.2\times10-\frac{1}{2}g\times10^2$ **OR** $-39.2\times10+\frac{1}{2}g\times10^2$	A1	1.1b
	98 (m) (must be positive)	A1	1.1b
		(3)	
2(b)	Complete method to find either half the time or the full time	M1	3.1b
	Correct equation e.g. $0=24.5-gt$ **OR** $-24.5=24.5-gt$	A1	1.1b
	5 (s)	A1	1.1b
		(3)	
2(c)	e.g. (include) air resistance	B1	3.5c
		(1)	
			(7 marks)

1. A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of $3.2\,\mathrm{m\,s^{-2}}$

Find

(a) the speed of the car after 5 s,

(1)

(b) the distance travelled by the car in the first 5 s.

(2)

Constant Accn. SUVAT

1(a)	16 ($\mathrm{m\,s^{-1}}$) seen as the answer	B1
		(1)
1(b)	$s=\frac{1}{2}\times 3.2\times 5^2$ **OR** $s=\frac{(0+16)}{2}\times 5$ **OR** $s=(16\times 5)-\frac{1}{2}\times 3.2\times 5^2$ **OR** $16^2=2\times 3.2\times s$ **OR** from a *v*-*t* graph, $s=\frac{1}{2}\times 5\times 16$	M1
	s = 40 (m)	A1
		(2)

Variable Acceleration

8. A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leqslant t \leqslant 10$$

(a) Explain the restriction, $0 \leqslant t \leqslant 10$

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

Question	Scheme	Marks	AOs
8(a)	Substitution of both $t = 0$ and $t = 10$	M1	2.1
	$s = 0$ for both $t = 0$ and $t = 10$	A1	1.1b
	Explanation ($s > 0$ for $0 < t < 10$) since $s = \frac{1}{10}t^2(t-10)^2$	A1	2.4
		(3)	
(b)	Differentiate displacement s w.r.t. t to give velocity, v	M1	1.1a
	$v = \frac{1}{10}(4t^3 - 60t^2 + 200t)$	A1	1.1b
	Interpretation of 'rest' to give $v = \frac{1}{10}(4t^3 - 60t^2 + 200t) = \frac{2}{5}t(t-5)(t-10) = 0$	M1	1.1b
	$t = 0, 5, 10$	A1	1.1b
	Select $t = 5$ and substitute their $t = 5$ into s	M1	1.1a
	Distance $= 62.5$ m	A1 ft	1.1b
		(6)	

8. A particle, P, moves along the x-axis. At time t seconds, $t \geqslant 0$, the displacement, x metres, of P from the origin O, is given by $x = \frac{1}{2}t^2(t^2 - 2t + 1)$

(a) Find the times when P is instantaneously at rest.

(5)

(b) Find the total distance travelled by P in the time interval $0 \leqslant t \leqslant 2$

(3)

(c) Show that P will never move along the negative x-axis.

(2)

Question	Scheme	Marks	AOs
8(a)	Multiply out and differentiate *wrt* to time (or use of product rule i.e. must have two terms with correct structure)	M1	1.1a
	$v = 2t^3 - 3t^2 + t$	A1	1.1b
	$2t^3 - 3t^2 + t = 0$ and solve: $t(2t-1)(t-1) = 0$	DM1	1.1b
	$t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two	A1	1.1b
	All three	A1	1.1b
		(5)	
(b)	Find x when $t = 0, \frac{1}{2}$, 1 and 2 : $(0, \frac{1}{32}, 0, 2)$	M1	2.1
	Distance = $\frac{1}{32} + \frac{1}{32} + 2$	M1	2.1
	$2\frac{1}{16}$ (m) oe or 2.06 or better	A1	1.1b
		(3)	
(c)	$x = \frac{1}{2}t^2(t-1)^2$	M1	3.1a
	$\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative	A1 cso	2.4
		(2)	
		(10 marks)	

HOME

3. A particle, P, moves along a straight line such that at time t seconds, $t \geqslant 0$, the velocity of P, $v\,\text{m}\,\text{s}^{-1}$, is modelled as

$$v = 12 + 4t - t^2$$

Find

(a) the magnitude of the acceleration of P when P is at instantaneous rest,

(5)

(b) the distance travelled by P in the interval $0 \leqslant t \leqslant 3$

(3)

AS 2019

Variable Acceleration

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$v = 12 + 4t - t^2 = 0$	M1	This mark is give for setting the equation for v equal to zero
	$v = (6 - t)(2 + t) = 0$ $t = 6$	A1	This mark is given for solving to find t
	$a = \frac{dv}{dt}$	M1	This mark is given for differentiating v with respect to t to find the acceleration
	$a = 4 - 2t$	A1	This mark is given for finding a correct expression for a
	When $t = 6$, $a = -8$ The magnitude of the acceleration is 8	A1	This mark is given for finding a correct value for the magnitude of the acceleration
(b)	$s = \int 12 + 4t - t^2 \; dt$	M1	This mark is given for integrating v with respect to t to find the distance
	$s = 12t + 2t^2 - \frac{1}{3}t^3 \; (+c)$	A1	This mark is given for a correct integral for v
	$\left[12t + 2t^2 - \frac{1}{3}t^3\right]_0^3 = 45$ (m)	A1	This mark is given for a correct evaluation from 0 to 3 to find the distance travelled

HOME

3. A particle P moves along a straight line such that at time t seconds, $t \geqslant 0$, after leaving the point O on the line, the velocity, $v\,\text{m s}^{-1}$, of P is modelled as

$$v = (7 - 2t)(t + 2)$$

(a) Find the value of t at the instant when P stops accelerating.

(4)

(b) Find the distance of P from O at the instant when P changes its direction of motion.

(5)

In this question, solutions relying on calculator technology are not acceptable.

Variable Acceleration

3(a)	$v = 3t - 2t^2 + 14$ and differentiate	M1	3.1a
	$a = \frac{dv}{dt} = 3 - 4t$ or $(7 - 2t) - 2(t + 2)$ using product rule	A1	1.1b
	$3 - 4t = 0$ and solve for t	M1	1.1b
	$t = \frac{3}{4}$ oe	A1	1.1b
		(4)	
3(b)	Solve problem using $v = 0$ to find a value of t $\left(t = \frac{7}{2}\right)$	M1	3.1a
	$v = 3t - 2t^2 + 14$ and integrate	M1	1.1b
	$s = \frac{3t^2}{2} - \frac{2t^3}{3} + 14t$	A1	1.1b
	Substitute $t = \frac{7}{2}$ into their s expression (M0 if using *suvat*)	M1	1.1b
	$s = \frac{931}{24} = 38\frac{19}{24} = 38.79166..(\text{m})$ Accept 39 or better	A1	1.1b
		(5)	
			(9 marks)

2. A particle P moves along a straight line.

At time t seconds, the velocity $v\,\mathrm{m\,s^{-1}}$ of P is modelled as

$$v = 10t - t^2 - k \qquad t \geqslant 0$$

where k is a constant.

(a) Find the acceleration of P at time t seconds.

(2)

The particle P is instantaneously at rest when $t = 6$

(b) Find the other value of t when P is instantaneously at rest.

(4)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 6$

(4)

AS 2021

Variable Acceleration

Question	Scheme	Marks	AOs
2(a)	Differentiate v w.r.t. t	M1	3.1a
	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 2t$ isw	A1	1.1b
		(2)	
2(b)	Solve problem using $v = 0$ when $t = 6$	M1	3.1a
	$0 = 10t - t^2 - 24$	A1	1.1b
	Solve quadratic oe to find other value of t	M1	1.1b
	$t = 4$	A1	1.1b
		(4)	
2(c)	Integrate v or $-v$ w.r.t. t	M1	3.1a
	$5t^2 - \frac{1}{3}t^3 - 24t$	A1	1.1b
	Total distance $= -\left[5t^2 - \frac{1}{3}t^3 - 24t\right]_0^4 + \left[5t^2 - \frac{1}{3}t^3 - 24t\right]_4^6$	M1	2.1
	$\frac{116}{3}$ (m)	A1	1.1b
		(4)	
		(10 marks)	

HOME

3. A fixed point O lies on a straight line.

A particle P moves along the straight line.

At time t seconds, $t \geqslant 0$, the distance, s metres, of P from O is given by

$$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$$

(a) Find the acceleration of P at each of the times when P is at instantaneous rest.

(6)

(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(3)

3(a)	Differentiate s wrt t	M1	3.1a
	$(v=)\ t^2-5t+6$	A1	1.1b
	Equate their v to 0 and solve	M1	1.1b
	$t=2$ or 3	A1	1.1b
	$(a=)\ 2t-5$	B1ft	2.1
	$a=$ 1 and -1 (m s^{-2}) isw (A0 if extras)	A1	1.1b
		(6)	
(b)	Attempt to find values of s for $t=2, 3$ *and* 4 oe Correct values are $\left(s_2=\frac{14}{3}, s_3=\frac{9}{2} \text{ and } s_4=\frac{16}{3}\right)$ Could be implied by correct values for: s_2, (s_3-s_2) and (s_4-s_3) which are $\frac{14}{3}$, $(-\frac{1}{6})$ and $\frac{5}{6}$	**DM**1	1.1b
	Total distance travelled $=s_2+(s_2-s_3)+s_4-s_3$ **OR** $s_2-(s_3-s_2)+s_4-s_3$ **OR** $\left[\frac{1}{3}t^3-\frac{5}{2}t^2+6t\right]_0^2-\left[\frac{1}{3}t^3-\frac{5}{2}t^2+6t\right]_2^3+\left[\frac{1}{3}t^3-\frac{5}{2}t^2+6t\right]_3^4$ **OR** $\frac{14}{3}\ -\ (-\frac{1}{6})\ +\ \frac{5}{6}$ **OR** $s_2+2(s_2-s_3)+s_4-s_2$ $(=2s_2-2s_3+s_4)$ oe	M1	2.1
	$5\frac{2}{3}$ oe (m) Accept 5.7 or better	A1	1.1b

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A fixed point O lies on a straight line.

A particle P moves along the straight line such that at time t seconds, $t \geqslant 0$, after passing through O, the velocity of P, $v \text{ m s}^{-1}$, is modelled as

$$v = 15 - t^2 - 2t$$

(a) Verify that P comes to instantaneous rest when $t = 3$

(1)

(b) Find the magnitude of the acceleration of P when $t = 3$

(3)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(4)

Question	Scheme	Marks	AOs
3(a)	$15-3^2-2\times3=0$ *	B1*	1.1b
		(1)	
3(b)	Differentiate *v* wrt *t*	M1	2.1
	$-2t-2$	A1	1.1b
	$8\ (\mathrm{m\ s^{-2}})$	A1	1.1b
		(3)	
3(c)	Integrate *v* w.r.t. *t*	M1	1.1b
	$15t-\frac{1}{3}t^3-t^2$	A1	1.1b
	Total distance = $\left[15t-\frac{1}{3}t^3-t^2\right]_0^3-\left[15t-\frac{1}{3}t^3-t^2\right]_3^4$ **OR** $s_3+(s_3-s_4)$ where s_3 means the value of their integral when t = 3. **N.B.** Allow the negative of this.	M1	3.1a
	$\frac{94}{3}$ (m)	A1	1.1b
		(4)	
		(8 marks)	

HOME

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A particle is moving along a straight line.

At time t seconds, $t > 0$, the velocity of the particle is $v \text{ m s}^{-1}$, where

$$v = 2t - 7\sqrt{t} + 6$$

(a) Find the acceleration of the particle when $t = 4$

(3)

When $t = 1$ the particle is at the point X.

When $t = 2$ the particle is at the point Y.

Given that the particle does not come to instantaneous rest in the interval $1 < t < 2$

(b) show that $XY = \frac{1}{3}\left(41 - 28\sqrt{2}\right)$ metres.

(4)

Variable Acceleration

2(a)	Differentiate $2t-7\sqrt{t}+6$ wrt t	M1	3.1a
	$2-\frac{7}{2\sqrt{t}}$ oe	A1	1.1b
	When $t=4$, $a=0.25\,(\text{ms}^{-2})$	A1	1.1b
		(3)	
(b)	Integrate $2t-7\sqrt{t}+6$ wrt t	M1	3.1a
	$t^2-\frac{14}{3}t^{\frac{3}{2}}+6t(+C)$	A1	1.1b
	Use the limits to find XY	DM1	1.1b
	$(XY=)\ \frac{1}{3}\left(41-28\sqrt{2}\right)$ (metres) *	A1*	1.1b
		(4)	

(7 marks)

Forces

4.

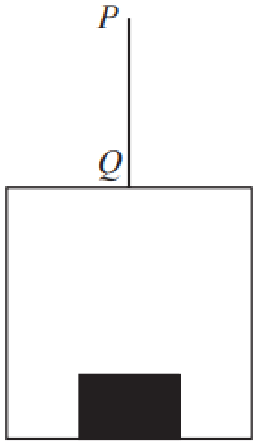


Figure 1

A vertical rope PQ has its end Q attached to the top of a small lift cage.

The lift cage has mass 40 kg and carries a block of mass 10 kg, as shown in Figure 1.

The lift cage is raised vertically by moving the end P of the rope vertically upwards with constant acceleration $0.2\,\text{m s}^{-2}$

The rope is modelled as being light and inextensible and air resistance is ignored.

Using the model,

(a) find the tension in the rope PQ

(3)

(b) find the magnitude of the force exerted on the block by the lift cage.

(3)

Question	Scheme	Marks
	N.B. Use the mass in the '*ma*' term of an equation to determine which part of the system (cage and block, cage or block) it applies to.	
4(a)	Translate situation into the model and set up the **equation** of motion for the cage and the block to obtain an equation in T only.	M1
	$T-40g-10g=50\times0.2$	A1
	500 (N) Must be positive	A1
	Some examples: $T-50=50\times0.2$ and $T-40g-10g=50g\times0.2$ both score M1A0A0	
		(3)
(b)	Use the model to set up the equation of motion for the block to obtain an equation in R only.	M1
	$R-10g=10\times0.2$ Allow $-R$ instead of R	A1
	100 (N) Must be positive.	A1
	OR: Use the model to set up the equation of motion for the cage to obtain an equation in R only.	M1
	$T-40g-R=40\times0.2$ with their T substituted	A1
	100 (N) Must be positive	A1
		(3)

3. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle P is moving on a smooth horizontal surface under the action of two forces.

Given that

- the mass of P is 2 kg
- the two forces are $(2\mathbf{i} + 4\mathbf{j})$ N and $(c\mathbf{i} - 2\mathbf{j})$ N, where c is a constant
- the magnitude of the acceleration of P is $\sqrt{5}\,\mathrm{m\,s^{-2}}$

find the two possible values of c.

(5)

3	Resultant force $=(2+c)\mathbf{i}+(4-2)\mathbf{j}$	B1	1.1b
	$\sqrt{(2+c)^2+2^2}$ or $(2+c)^2+2^2$	M1	3.1a
	$(2+c)^2+4=4\times5$ or $\sqrt{(2+c)^2+2^2}=2\times\sqrt{5}$	M1	3.1a
	OR		
	$\mathbf{a}=\frac{1}{2}\left[(2+c)\mathbf{i}+(4-2)\mathbf{j}\right]$ oe	M1	
	$\frac{1}{4}\left[(2+c)^2+4\right]=5$ or $\frac{1}{2}\sqrt{(2+c)^2+2^2}=\sqrt{5}$	M1	
	$c=2$ or $c=-6$	A1	1.1b
	$c=2$ and $c=-6$	A1	2.2a
		(5)	

(5 marks)

Connected Particles

Connected Particles

A small ball A of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley P which is fixed at the edge of the table. The other end of the string is attached to a small ball B of mass 1.5 kg hanging freely, vertically below P and with B at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of A from the rough table is modelled as having constant magnitude 12.7 N. Ball B reaches the floor before ball A reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible and the pulley is modelled as being small and smooth.

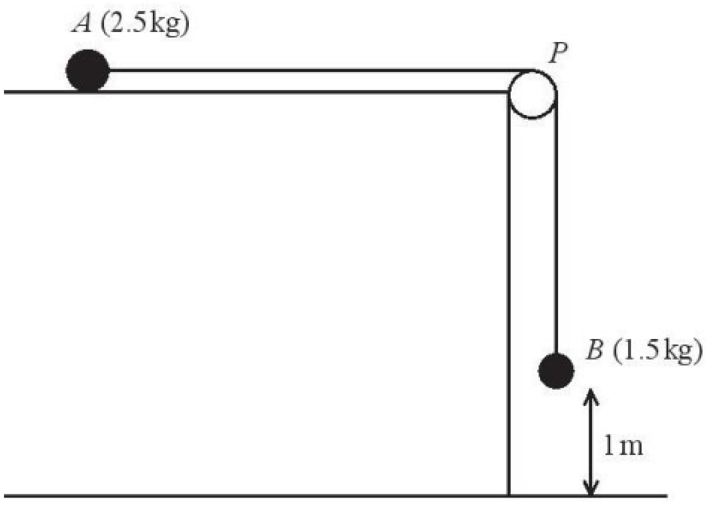


Figure 2

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence find the acceleration of B.

(2)

(c) Using the model, find the time it takes, from release, for B to reach the floor.

(2)

It was found that it actually took 2.3 seconds for ball B to reach the floor.

(d) Using this information

(i) comment on the appropriateness of using the model to find the time it takes ball B to reach the floor, justifying your answer.

(ii) suggest one improvement that could be made in the model.

(2)

Question	Scheme	Marks	AOs
9(a) (i)	Equation of motion for *A*	M1	3.3
	$T - 12.7 = 2.5a$	A1	1.1b
(ii)	Equation of motion for *B*	M1	3.3
	$1.5g - T = 1.5a$	A1	1.1b
		(4)	
(b)	Solving two equations for *a*	M1	1.1b
	$a = 0.5$	A1	1.1b
		(2)	
(c)	$1 = \frac{1}{2} \leftarrow 0.5\, t^2$	M1	3.4
	$t = 2$ seconds	A1ft	1.1b
		(2)	
(d)	(i) Not very appropriate for valid reason, see below in notes	**B1**	3.5a
	(ii) Valid improvement in model, see below in notes.	**B1**	3.5c
		(2)	
			(10 marks)

HOME

AS 2018

Connected Particles

Two small balls, P and Q, have masses $2m$ and km respectively, where $k < 2$.
The balls are attached to the ends of a string that passes over a fixed pulley.
The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 1.

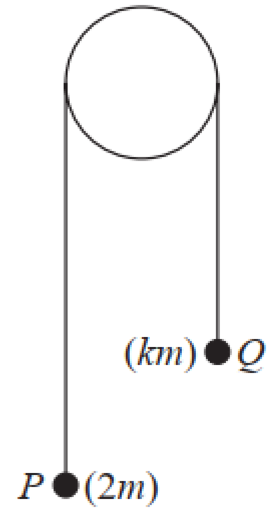


Figure 1

The system is released from rest and, in the subsequent motion, P moves downwards with an acceleration of magnitude $\frac{5g}{7}$

The balls are modelled as particles moving freely.
The string is modelled as being light and inextensible.
The pulley is modelled as being small and smooth.

Using the model,

(a) find, in terms of m and g, the tension in the string,

(3)

(b) explain why the acceleration of Q also has magnitude $\frac{5g}{7}$

(1)

(c) find the value of k.

(4)

(d) Identify one limitation of the model that will affect the accuracy of your answer to part (c).

(1)

Question	Scheme	Marks	AOs
9(a)	Equation of motion for *P*	M1	3.3
	$2mg - T = 2m \leftrightarrow \frac{5g}{7}$	A1	1.1b
	$T = \frac{4mg}{7}$	A1	1.1b
		(3)	
(b)	Since the string is modelled as being inextensible	B1	3.4
		(1)	
(c)	Equation of motion for *Q* **OR** for whole system	M1	3.3
	$T - kmg = km \leftrightarrow \frac{5g}{7}$ **OR** $2mg - kmg = (km + 2m)\frac{5g}{7}$	A1	1.1b
	$\frac{4mg}{7} - kmg = km \leftrightarrow \frac{5g}{7}$ oe and <u>solve for *k*</u>	DM1	1.1b
	$k = \frac{1}{3}$ or 0.333 or better	A1	1.1b
		(4)	
(d)	e.g The model does not take account of the mass of the string (SEE BELOW for alternatives)	B1	3.5b
		(1)	
	(9 marks)		

HOME

A small ball, P, of mass 0.8 kg, is held at rest on a smooth horizontal table and is attached to one end of a thin rope.

The rope passes over a pulley that is fixed at the edge of the table.

The other end of the rope is attached to another small ball, Q, of mass 0.6 kg, that hangs freely below the pulley.

Ball P is released from rest, with the rope taut, with P at a distance of 1.5 m from the pulley and with Q at a height of 0.4 m above the horizontal floor, as shown in Figure 1.

Ball Q descends, hits the floor and does not rebound.

The balls are modelled as particles, the rope as a light and inextensible string and the pulley as small and smooth.

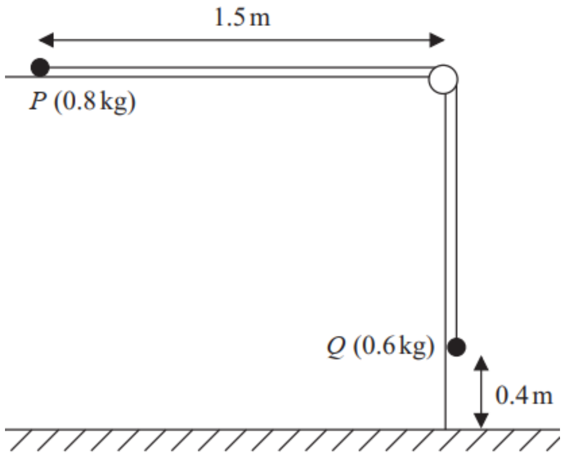


Figure 1

Using this model,

(a) show that the acceleration of Q, as it falls, is $4.2\,\text{m}\,\text{s}^{-2}$ **(5)**

(b) find the time taken by P to hit the pulley from the instant when P is released. **(6)**

(c) State one limitation of the model that will affect the accuracy of your answer to part (a). **(1)**

HOME

(a)	$0.6g - T$	M1	This mark is given for a method to find an equation of motion for Q
	$0.6g - T = 0.6a$	A1	This mark is given for a correct equation of motion for Q
	$P = -T + 0.8a = 0$	M1	This mark is given for a correct equation of motion for P
	$T = 0.8a$	A1	This mark is given for finding a correct value for T
	$0.6g - 0.8a = 0.6a$ $a = \frac{0.6g}{1.4} = \frac{5.88}{1.4}$ $a = 4.2$ (m s^{-2})	A1	This mark is given for finding a correct value for the acceleration of Q
(b)	$0.4 = \frac{1}{2} \times 4.2 \times t_1^2$	M1	This mark is given using $s = \frac{1}{2}at^2$ to find the time for Q to hit the floor
	$t_1 = 0.436$	A1	This mark is given for solving to find t_1 correctly
	$v = 0 + 4.2 \times 0.436$ or $v = \sqrt{2 \times 4.2 \times 0.4}$	M1	This mark is given for using $v = u + at$ or $v^2 = 2as$ to find the speed of P
	$t_2 = \frac{1.5 - 0.4}{v}$	M1	This mark is given for a method to find the time for P to hit the pulley after Q hits the floor
	$t_1 + t_2 = 0.436 + \frac{1.5 - 0.4}{1.8312}$	M1	This mark is given for a method to find the time taken by summing t_1 and t_2
	1.04 s	A1	This mark is given for finding the time taken by P to hit the pulley
(c)	For example: The rope is light The rope in inextensible The pulley is smooth	B1	This mark is given for a valid limitation stated

2.

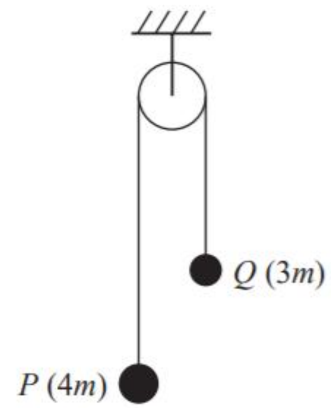


Figure 1

One end of a string is attached to a small ball P of mass $4m$.

The other end of the string is attached to another small ball Q of mass $3m$.

The string passes over a fixed pulley.

Ball P is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 1.

Ball P is released.

The string is modelled as being light and inextensible, the balls are modelled as particles, the pulley is modelled as being smooth and air resistance is ignored.

(a) Using the model, find, in terms of m and g, the magnitude of the force exerted on the pulley by the string while P is falling and before Q hits the pulley.

(8)

(b) State one limitation of the model, apart from ignoring air resistance, that will affect the accuracy of your answer to part (a).

(1)

2(a)	Equation of motion for P with usual rules	M1	3.3
	$4mg - T = 4ma$	A1	1.1b
	Equation of motion for Q with usual rules	M1	3.3
	$T - 3mg = 3ma$	A1	1.1b
	Solve these equations for T (does not need to be in terms of mg)	M1	1.1b
	$T = \frac{24mg}{7}$ in any form (does not need to be a single term)	A1	1.1b
	Force on pulley = $2T$	M1	3.4
	$\frac{48mg}{7}$ Accept $6.9mg$ or better	A1	1.1b
		(8)	
2(b)	Weight of the rope or extensibility of rope Or: pulley may not be smooth	B1	3.5b
		(1)	

(9 marks)

A ball P of mass $2m$ is attached to one end of a string.

The other end of the string is attached to a ball Q of mass $5m$.

The string passes over a fixed pulley.

The system is held at rest with the balls hanging freely and the string taut.

The hanging parts of the string are vertical with P at a height $2h$ above horizontal ground and with Q at a height h above the ground, as shown in Figure 1.

The system is released from rest.

In the subsequent motion, Q does not rebound when it hits the ground and P does not hit the pulley.

The balls are modelled as particles.

The string is modelled as being light and inextensible.

The pulley is modelled as being small and smooth.

Air resistance is modelled as being negligible.

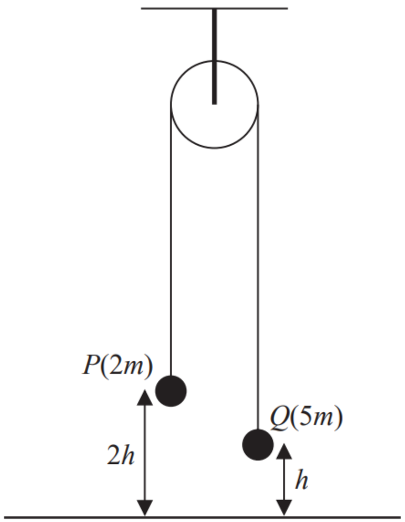


Figure 1

Using this model,

(a) (i) write down an equation of motion for P,

(ii) write down an equation of motion for Q,

(4)

(b) find, in terms of h only, the height above the ground at which P first comes to instantaneous rest.

(7)

(c) State one limitation of modelling the balls as particles that could affect your answer to part (b).

(1)

In reality, the string will not be inextensible.

(d) State how this would affect the accelerations of the particles.

(1)

Question	Scheme	Marks	AOs
3(a)	(i) Equation of motion for P	M1	3.3
	$T - 2mg = 2ma$	A1	1.1b
	(ii) Equation of motion for Q	M1	3.3
	$5mg - T = 5ma$	A1	1.1b
	N.B. (allow ($-a$) in both equations)	**(4)**	
3(b)	Solve equations for a or use whole system equation and solve for a	M1	3.4
	$a = \frac{3g}{7} = 4.2$	A1	1.1b
	$v = \sqrt{2 \times \frac{3g}{7} \times h} = \sqrt{8.4h}$ or $v^2 = 2 \times \frac{3g}{7} \times h$ $(= 8.4h)$	M1	1.1b
	$0 = \frac{6gh}{7} - 2gH$	M1	1.1b
	$H = \frac{3h}{7}$	A1	1.1b
	Total height $= 2h + h + H$	M1	2.1
	Total height $= \frac{24h}{7}$	A1	1.1b
		(7)	
3(c)	e.g. The distance that Q falls to the ground would not be exactly h oe	B1	3.5b
		(1)	
3(d)	e.g. The accelerations of the balls would not have equal magnitude (allow 'wouldn't be the same' oe) B0 if they say 'inextensible => acceleration same'	B1	3.5a
		(1)	
		(13 marks)	

4.

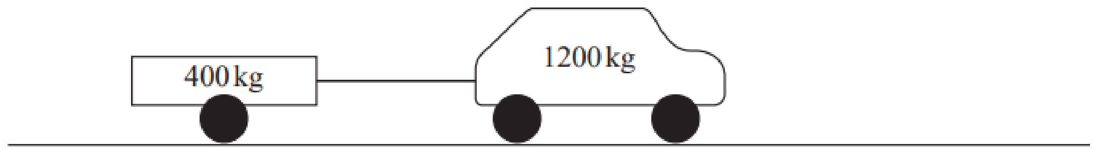


Figure 2

A car of mass 1200 kg is towing a trailer of mass 400 kg along a straight horizontal road using a tow rope, as shown in Figure 2.
The rope is horizontal and parallel to the direction of motion of the car.

- The resistance to motion of the car is modelled as a constant force of magnitude $2R$ newtons
- The resistance to motion of the trailer is modelled as a constant force of magnitude R newtons
- The rope is modelled as being light and inextensible
- The acceleration of the car is modelled as $a\,\text{m s}^{-2}$

The driving force of the engine of the car is 7400 N and the tension in the tow rope is 2400 N.

Using the model,

(a) find the value of a

(5)

In a refined model, the rope is modelled as having mass and the acceleration of the car is found to be $a_1\,\text{m s}^{-2}$

(b) State how the value of a_1 compares with the value of a

(1)

(c) State one limitation of the model used for the resistance to motion of the car.

(1)

HOME

4(a)	Equation of motion for the car	M1
	$7400 - 2R - 2400 = 1200a$	A1
	Equation of motion for the trailer	M1
	$2400 - R = 400a$	A1
	$a = 0.5$	A1
		(5)
	N.B. Either equation could be replaced by: Equation of motion for the whole system $7400 - 3R = 1600a$	
4(b)	The value of a_1 would be less than the value of a. Allow 'a_1 would be slower than a', **N.B.** Allow 'it would be less than a'	B1
		(1)
4(c)	The resistance won't be constant or just 'it won't be constant.' Allow the negative also: The resistance is constant or just 'it is constant' B0 for 'it doesn't take account of air resistance'	B1
		(1)

Figure 2 shows a car towing a trailer along a straight horizontal road.

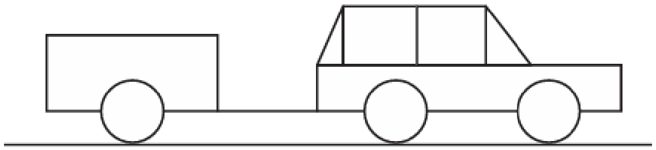


Figure 2

The mass of the car is 800 kg and the mass of the trailer is 600 kg.

The trailer is attached to the car by a towbar which is parallel to the road and parallel to the direction of motion of the car and the trailer.

The towbar is modelled as a light rod.

The resistance to the motion of the car is modelled as a constant force of magnitude 400 N.

The resistance to the motion of the trailer is modelled as a constant force of magnitude *R* newtons.

The engine of the car is producing a constant driving force that is horizontal and of magnitude 1740 N.

The acceleration of the car is $0.6\,\mathrm{m\,s^{-2}}$ and the tension in the towbar is *T* newtons.

Using the model,

(a) show that $R = 500$

(3)

(b) find the value of *T*.

(3)

At the instant when the speed of the car and the trailer is $12.5\,\mathrm{m\,s^{-1}}$, the towbar breaks.

The trailer moves a further distance *d* metres before coming to rest.

The resistance to the motion of the trailer is modelled as a constant force of magnitude 500 N.

Using the model,

(c) show that, after the towbar breaks, the deceleration of the trailer is $\frac{5}{6}\,\mathrm{m\,s^{-2}}$

(1)

(d) find the value of *d*.

(3)

In reality, the distance *d* metres is likely to be different from the answer found in part (d).

(e) Give two **different** reasons why this is the case.

(2)

AS 2024

Connected Particles

4(a)	Equation of motion:	M1	3.3
	$1740-400-R=(600+800)\times 0.6$ Or $\begin{cases}1740-T-400=800\times 0.6\\ T-R=600\times 0.6\end{cases}$ with T eliminated or found (860) from the first equation, and then used in the second to find R.	A1	1.1b
	$R=1740-840-400=500$ *	A1*	2.2a
		(3)	
(b)	Equation of motion for car or trailer	M1	3.4
	$1740-T-400=800\times 0.6$ or $T-500=600\times 0.6$	A1	1.1b
	$T=860$	A1	1.1b
		(3)	
(c)	Use of $500=\pm 600a$ to obtain * **N.B.** Need to see explicitly deceleration $=\frac{5}{6}$	B1*	1.1b
		(1)	
(d)	Complete method to find distance with $a=\pm\frac{5}{6}$	M1	3.4
	$0=12.5^2-2\times\frac{5}{6}\times d$ **OR e.g.** $t=\frac{12.5}{\frac{5}{6}}=15$ then $d=\frac{1}{2}\times\frac{5}{6}\times 15^2$ or $d=\frac{1}{2}\times(0+12.5)\times 15$ or $d=12.5\times 15+\frac{1}{2}\times(-\frac{5}{6})\times 15^2$	A1	1.1b
	93.75 oe	A1	1.1b
		(3)	
(e)	**N.B.** If **more than two** answers given, subtract 1 from any marks earned for each incorrect extra answer which are in group 7 below but do **not** penalise answers which are in group 8 and then, on ePEN, award as appropriate either: B1B1, B1B0 or B0B0 but NOT B0B1.	B1 B1	3.5b 3.5b
		(2)	

Correct answers (not verbatim but equivalent to)

1.
Resistance to motion of the trailer will be different (when not in the slipstream of the car).
or there will be more air or wind resistance.
Resistance to motion of the trailer will not be constant/ be exactly 500 N.
Wind or air resistance would not be constant.
The deceleration won't be constant/ be exactly 5/6 m s^{-2} .

2.
The model takes no account of forces acting side to side.
The trailer may not continue to move in a straight line.

3.
The trailer could be affected by any unevenness of the road e.g. potholes, bumps etc
Does not take account of the type of ground.

4.
Not considered the mass of the towbar.

5.
Trailer emergency brake may engage.

6.
After the towbar breaks the trailer will tip and drag on the road.
The trailer will be unstable.

Incorrect answers which incur a penalty (not verbatim but equivalent to).

7.
Any answer which mentions the car.
The acceleration wouldn't be equal.
Not considered the length of the trailer.
Road might not be straight and/or horizontal.
Mass of the trailer.
Does not take friction (between the tyres and the ground) into account.
Does not take air resistance into account.
Does not take wind resistance into account.

Incorrect answers which do NOT incur a penalty (not verbatim but equivalent to).

8.
Obstacles in the road or cars which the trailer could hit.
Does not take account of weather conditions e.g. wind, rain, snow etc
The dimensions/shape of the trailer would slow it down.
It won't be travelling at a constant speed oe.

Distance/Speed Time Graphs

6.

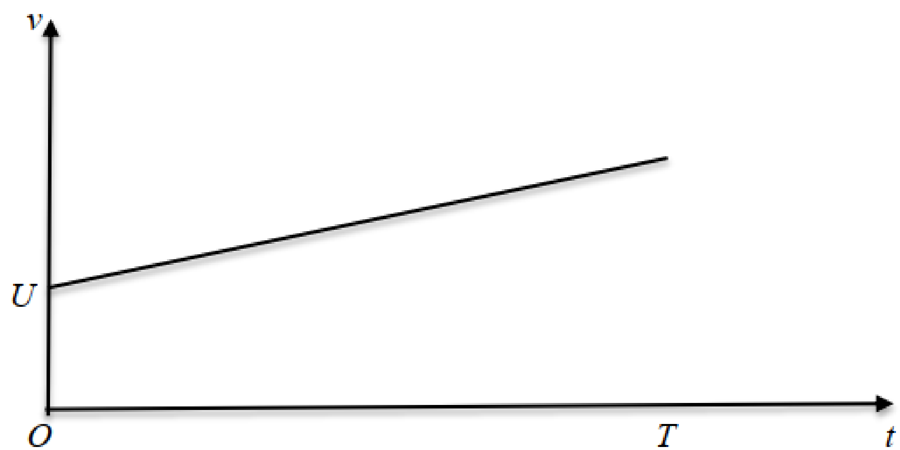


Figure 1

A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is U m s^{-1}. The car then accelerates with constant acceleration a m s^{-2} for T seconds. The car travels a distance D metres during these T seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \leqslant t \leqslant T$.

Using the graph, show that $D = UT + \frac{1}{2}aT^2$.

(No credit will be given for answers which use any of the kinematics (*suvat*) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

(4)

Question	Scheme	Marks	AOs
6.	Using distance = total area under graph (e.g. area of rectangle + triangle or trapezium or rectangle – triangle)	M1	2.1
	e.g. $D = UT + \frac{1}{2} Th$, where h is height of triangle	A1	1.1b
	Using gradient = acceleration to substitute $h = aT$	M1	1.1b
	$D = UT + \frac{1}{2} aT^2$ *	A1 *	1.1b
		4	
			(4 marks)

AS 2018

Dist/Speed Time Graphs

7. A train travels along a straight horizontal track between two stations, A and B.

In a model of the motion, the train starts from rest at A and moves with constant acceleration $0.3\,\text{m s}^{-2}$ for 80 s.
The train then moves at constant velocity before it moves with a constant deceleration of $0.5\,\text{m s}^{-2}$, coming to rest at B.

(a) For this model of the motion of the train between A and B,

(i) state the value of the constant velocity of the train,

(ii) state the time for which the train is decelerating,

(iii) sketch a velocity-time graph.

(3)

The total distance between the two stations is 4800 m.

(b) Using the model, find the total time taken by the train to travel from A to B.

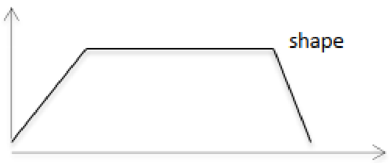
(3)

(c) Suggest one improvement that could be made to the model of the motion of the train from A to B in order to make the model more realistic.

(1)

AS 2018

Dist/Speed Time Graphs

7(a) (i)	24 ($m\ s^{-1}$)	B1	1.1b
(ii)	48 (s)	B1	1.1b
(iii)		B1	1.1b
		(3)	
(b)	Equating area under graph to 4800 to give equation in one unknown	M1	3.1b
	$\frac{1}{2}(T+T+80+48)\times 24 = 4800$ **OR** $(\frac{1}{2}\times 80\times 24)+24T+(\frac{1}{2}\times 48\times 24)=4800$ **oe**	A1ft	1.1b
	T = 136 so total time is 264 (s)	A1	1.1b
		(3)	
(c)	Accept **Either**: a smooth change from acceleration to constant velocity or from constant velocity to deceleration. **Or** have train accelerating and/or decelerating at a variable rate Do not accept e.g. Comments on air resistance or resistive forces, straightness of track, horizontal track, friction, length of train, mass of train, not having train moving with constant velocity. B0 if either an incorrect extra is included or an incorrect reason for a valid improvement is included. N.B. Variable acceleration due to air resistance is B0 **BUT** Variable acceleration due to **variable** air resistance is B1	B1	3.5c
		(1)	

HOME

1. At time $t = 0$, a parachutist falls vertically from rest from a helicopter which is hovering at a height of 550 m above horizontal ground.

The parachutist, who is modelled as a particle, falls for 3 seconds before her parachute opens.

While she is falling, and before her parachute opens, she is modelled as falling freely under gravity.

The acceleration due to gravity is modelled as being $10\,\text{m s}^{-2}$.

(a) Using this model, find the speed of the parachutist at the instant her parachute opens.

(1)

When her parachute is open, the parachutist continues to fall vertically.

Immediately after her parachute opens, she decelerates at $12\,\text{m s}^{-2}$ for 2 seconds before reaching a constant speed and she reaches the ground with this speed.

The total time taken by the parachutist to fall the 550 m from the helicopter to the ground is T seconds.

(b) Sketch a speed-time graph for the motion of the parachutist for $0 \leqslant t \leqslant T$.

(2)

(c) Find, to the nearest whole number, the value of T.

(5)

In a refinement of the model of the motion of the parachutist, the effect of air resistance is included before her parachute opens and this refined model is now used to find a new value of T.

(d) How would this new value of T compare with the value found, using the initial model, in part (c)?

(1)

(e) Suggest one further refinement to the model, apart from air resistance, to make the model more realistic.

(1)

AS 2019

Dist/Speed Time Graphs

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$V = 3\ (\text{s}) \times 10\ (\text{m s}^{-2}) = 30\ (\text{m s}^{-1})$	B1	This mark is given for finding the speed of the parachutist
(b)		B1	This mark is given for the correct shape of the graph
		B1	This mark is given for the correct numbers on the graph
(c)	$s = \frac{1}{2}(u+v)t + s = \frac{1}{2}(u+v)t + vT$	M1	This mark is given for a method to use the distance travelled to set up an equation in T only
	$550 =$ $\frac{1}{2}(0+30)\times 3 + \frac{1}{2}(30+6)\times 2 + 6(T-5)$	A1	These marks are given for a fully correct equation
		A1	
	$45 + 36 + 6T - 30 = 550$ $6T = 499$	M1	This mark is given for a method to solve the equation in T
	$T = 83$	A1	This mark is given for a correct value of T
(d)	The new value of T would be greater	B1	This mark is given for a correct conclusion
(e)	Allow for the effect of wind Allow for the dimensions of the parachutist and spin Use a more accurate version of g Allow that the parachutist doesn't fall vertically	B1	This mark is given for a valid refinement stated

HOME

2. A train travels along a straight horizontal track from station P to station Q.

In a model of the motion of the train, at time $t = 0$ the train starts from rest at P, and moves with constant acceleration until it reaches its maximum speed of $25\,\text{m s}^{-1}$

The train then travels at this constant speed of $25\,\text{m s}^{-1}$ before finally moving with constant deceleration until it comes to rest at Q.

The time spent decelerating is four times the time spent accelerating.

The journey from P to Q takes 700 s.

Using the model,

(a) sketch a speed-time graph for the motion of the train between the two stations P and Q.
(1)

The distance between the two stations is 15 km.

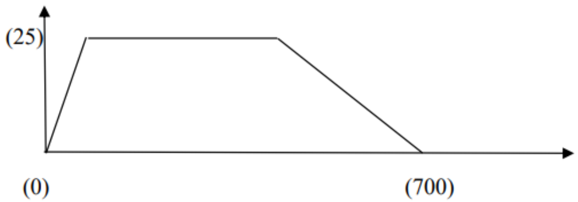
Using the model,

(b) show that the time spent accelerating by the train is 40 s,
(3)

(c) find the acceleration, in m s^{-2}, of the train,
(1)

(d) find the speed of the train 572 s after leaving P.
(2)

(e) State one limitation of the model which could affect your answers to parts (b) and (c).
(1)

Question	Scheme		Marks	AOs
2(a)		shape	B1	
			(1)	
(b)	Using *total* area = 15000 to set up an *equation* in **one unknown** Or they may use *suvat* on one or more sections (but must still be considering *all* sections) Allow an attempt at a clear explicit verification using $t = 40$ e.g. the following would score M1A1A1*: $4 \times 40 = 160$ then $700 - 40 - 160 = 500$ $\frac{(700+500)}{2} \times 25 = 15000 = 15$ km Withhold A1* if they don't include = 15 km **N.B.** M0 if a single *suvat* formula is used for the whole journey.		M1	
	$\frac{1}{2}(700+700-t-4t)\times 25 = 15000$ **OR** $\frac{1}{2}\times 25\times t + 25(700-t-4t)+\frac{1}{2}\times 25\times 4t = 15000$		A1	
	$t = 40$ (s)*		A1*	1.1b
			(3)	

Question	Scheme	Marks	AOs
(c)	0.63 or 0.625 or $\frac{5}{8}$ oe (m s^{-2}) isw	B1	1.1b/ (2.2a)
		(1)	
(d)	Complete method to find the speed or velocity at $t = 572$ e.g $\pm\left(25-(32\times\frac{5}{32})\right)$ or $\pm\left(128\times\frac{5}{32}\right)$ oe	M1	3.1b
	20 (m s^{-1})	A1	1.1b
		(2)	
(e)	e.g. (the train) cannot instantaneously change acceleration, (the train) won't move with constant acceleration , (the train) won't move with constant speed Allow negatives of these:	B1	3.5b
	e.g. (The train) moving at constant speed, or just 'constant speed' or 'constant acceleration' (is a limitation of the model) Must be a limitation of the model, so friction or air resistance or size of train is B0. **N.B.** Ignore incorrect **reasons** following a correct answer.		
		(1)	

HOME

AS 2023

Dist/Speed Time Graphs

Two children, Pat (P) and Sam (S), run a race along a straight horizontal track.

Both children start from rest at the same time and cross the finish line at the same time.

In a model of the motion:

Pat accelerates at a constant rate from rest for 5 s until reaching a speed of 4 m s^{-1} and then maintains a constant speed of 4 m s^{-1} until crossing the finish line.

Sam accelerates at a constant rate of 1 m s^{-2} from rest until reaching a speed of $X\text{ m s}^{-1}$ and then maintains a constant speed of $X\text{ m s}^{-1}$ until crossing the finish line.

Both children take 27.5 s to complete the race.

The velocity-time graphs shown in Figure 1 describe the model of the motion of each child from the instant they start to the instant they cross the finish line together.

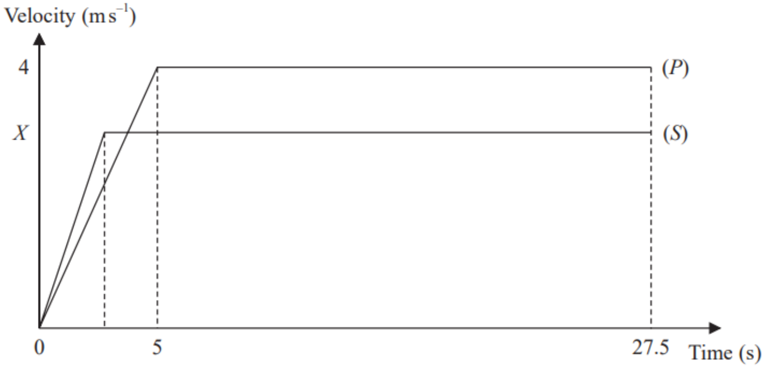


Figure 1

Using the model,

(a) explain why the areas under the two graphs are equal,

(1)

(b) find the acceleration of Pat during the first 5 seconds,

(1)

(c) find, in metres, the length of the race,

(2)

(d) find the value of X, giving your answer to 3 significant figures.

(4)

HOME

Question	Scheme	Marks
1(a)	Because the distances travelled or displacements are equal oe. If they mention the times are the same as well, ignore it.	B1
		(1)
1(b)	0.8 or 4/5 ($\mathrm{m\ s^{-2}}$)	B1
		(1)
1(c)	$\frac{1}{2}\times5\times4+(4\times22.5)$ **OR** $\frac{1}{2}(27.5+22.5)\times4$ **OR** $27.5\times4-\frac{1}{2}\times5\times4$	M1
	100 (m)	A1
		(2)
1(d)	Total area under graph = their answer for part (c)	M1
	$\frac{1}{2}X\times X+X(27.5-X)=100$	A1ft
	OR $\frac{1}{2}(27.5+27.5-X)\times X=100$ **OR** $27.5X-\frac{1}{2}X^2=100$	A1ft
	X = 3.92 to 3sf	A1
		(4)

1.

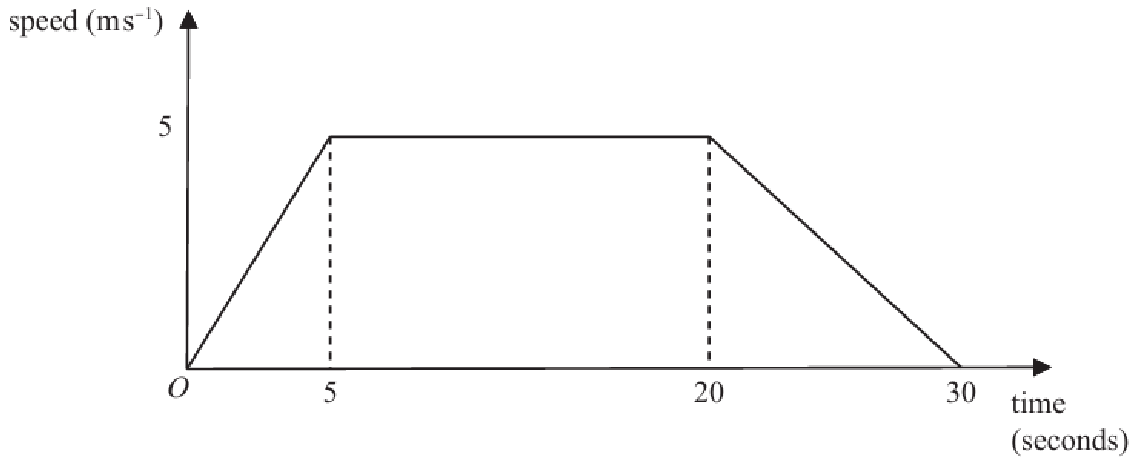


Figure 1

Figure 1 shows the speed-time graph for the journey of a car moving in a long queue of traffic on a straight horizontal road.

At time $t = 0$, the car is at rest at the point A.

The car then accelerates uniformly for 5 seconds until it reaches a speed of $5\,m\,s^{-1}$

For the next 15 seconds the car travels at a constant speed of $5\,m\,s^{-1}$

The car then decelerates uniformly until it comes to rest at the point B.

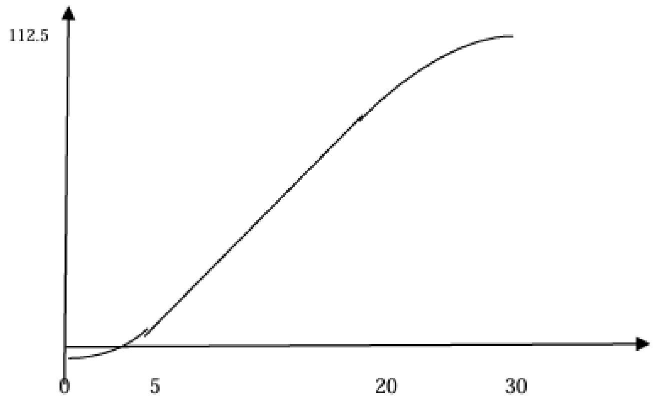
The total journey time is 30 seconds.

(a) Find the distance AB.

(3)

(b) Sketch a distance-time graph for the journey of the car from A to B.

(3)

1(a)	Complete method to find AB	M1	3.1b
	$=(\frac{1}{2}\times5\times5)+(5\times15)+(\frac{1}{2}\times5\times10)$ or $=\frac{1}{2}(30+15)\times5$ or $=(\frac{1}{2}\times5\times5)+\frac{1}{2}(25+15)\times5$ or $=\frac{1}{2}(20+15)\times5+(\frac{1}{2}\times5\times10)$ or $=(30\times5)-(\frac{1}{2}\times5\times5)-(\frac{1}{2}\times5\times10)$	A1	1.1b
	$=112.5$ (m)	A1	1.1b
		(3)	
(b)		B1 B1 B1	1.1b 1.1b 1.1b
		(3)	
		(6 marks)	

2.

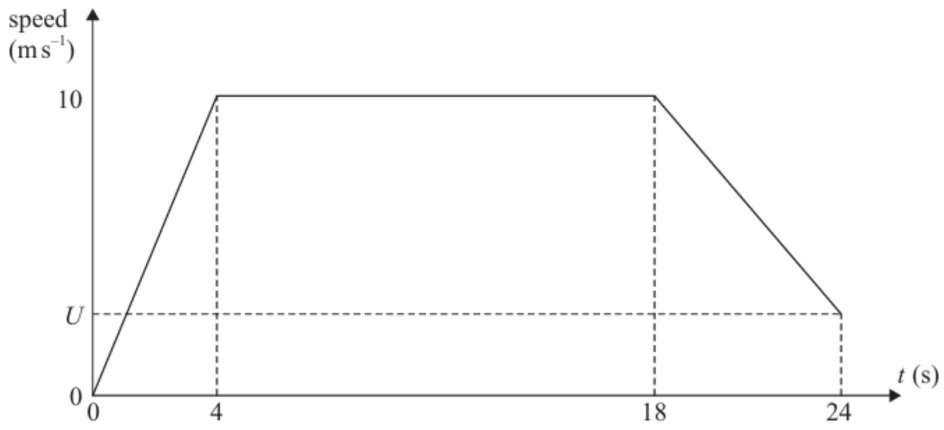


Figure 2

Figure 2 shows a speed-time graph for a model of the motion of an athlete running a **200 m** race in 24 s.

The athlete

- starts from rest at time $t = 0$ and accelerates at a constant rate, reaching a speed of $10\,\text{m s}^{-1}$ at $t = 4$
- then moves at a constant speed of $10\,\text{m s}^{-1}$ from $t = 4$ to $t = 18$
- then decelerates at a constant rate from $t = 18$ to $t = 24$, crossing the finishing line with speed $U\,\text{m s}^{-1}$

Using the model,

(a) find the acceleration of the athlete during the first 4 s of the race, stating the units of your answer,

(2)

(b) find the distance covered by the athlete during the first 18 s of the race,

(3)

(c) find the value of U.

(3)

2(a)	$\frac{10}{4}$	M1	3.4
	$2.5, \frac{5}{2}, \frac{10}{4}$ m s^{-2} units needed.	A1	1.1b
		(2)	
2(b)	Find the area, with correct structure, from $t = 0$ to 18	M1	3.1b
	$\frac{1}{2}\times 4\times 10+(14\times 10)$ triangle + rectangle **or** $\frac{1}{2}\times 10\times(14+18)$ trapezium **or** $(18\times 10)-\frac{1}{2}\times 4\times 10$ rectangle – triangle **N.B.** $\frac{1}{2}\times 4\times 10$ may be replaced by $\frac{1}{2}\times 2.5\times 4^2$ using $s = ut+\frac{1}{2}at^2$ or by $\frac{10^2-0^2}{2\times 2.5}$ using $v^2 = u^2+2as$	A1	1.1b
	160 (m)	A1	1.1b
		(3)	
2(c)	Using area, from $t = 18$ to $t = 24$, $= (200 - \text{their (b)})$ with correct structure **OR** $s = (200 - \text{their (b)})$, using *suvat* to find s **N.B.** If their (b) is incorrect and they don't use it, allow a correct restart.	M1	3.1b
	$6U+\frac{1}{2}\times 6\times(10-U) = 200-\text{their (b)}$ rectangle + triangle **or** $\frac{1}{2}\times 6\times(10+U) = 200-\text{their (b)}$ trapezium ($s = \left(\frac{u+v}{2}\right)t$) **or** $(6\times 10)-\frac{1}{2}\times 6\times(10-U) = 200-\text{their (b)}$ rectangle – triangle **or** $(10\times 6)+\frac{1}{2}\left(-\frac{(10-U)}{6}\right)\times 6^2 = 200-\text{their (b)}$ $s = ut+\frac{1}{2}at^2$ **or** $(U\times 6)-\frac{1}{2}\left(-\frac{(10-U)}{6}\right)\times 6^2 = 200-\text{their (b)}$ $s = vt-\frac{1}{2}at^2$ **N.B.** Two stage *suvat* method: $(10\times 6)+\frac{1}{2}a\times 6^2 = 200-\text{their (b)}$ => **AND** $U = 10+6\times\text{their } a$	A1ft	1.1b
	$\frac{10}{3} = 3\frac{1}{3}$ oe	A1	1.1b
		(3)	
			(8 marks)

HOME

Projectiles

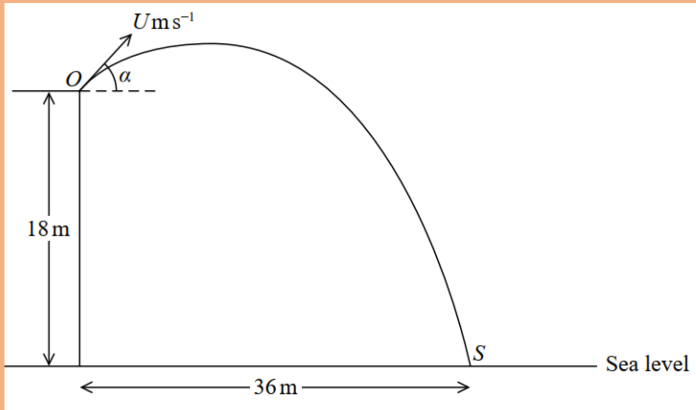


Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10$ m s^{-2}

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

A2 SAMs

Projectiles

Question	Scheme	Marks	AOs
10(a)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$36 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-18 = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	Correct strategy for solving the problem by setting up two equations in t and U and solving for U	M1	3.1b
	$U = 15$	A1	1.1b
		(6)	
(b)	Using the model and horizontal motion: $U\cos\alpha$ (12)	B1	3.4
	Using the model and vertical motion: $v^2 = (U\sin\alpha)^2 + 2(-10)(-7.2)$	M1	3.4
	$v = 15$	A1	1.1b
	Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1	3.1b
	$\sqrt{369} = 19$ m s^{-1} (2sf)	A1 **ft**	1.1b
		(5)	

(c)

B1, B1: for any two of

e.g. Include air resistance in the model of the motion

e.g. Use a more accurate value for g in the model of the motion

e.g. Include wind effects in the model of the motion

e.g. Include the dimensions of the stone in the model of the motion

10.

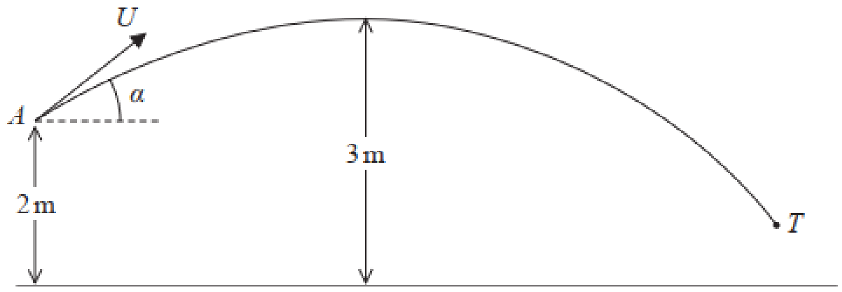


Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground. The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)

HOME

A2 2018

Projectiles

10(a)	Using the model and vertical motion: $0^2=(U\sin\alpha)^2-2g\times(3-2)$	M1	3.3
	$U^2=\dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s=ut$	M1	3.4
	$20=Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s=ut+\frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4}=Ut\sin\alpha-\frac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\frac{5}{4}=U\sin\alpha\left(\frac{20}{U\cos\alpha}\right)-\frac{1}{2}g\left(\frac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4}=20\tan\alpha-100\tan^2\alpha$		
	$(4\tan\alpha-1)(100\tan\alpha+5)=0$		
	$\tan\alpha=\frac{1}{4}\Rightarrow\alpha=14^0$ or better		

(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U=\sqrt{\dfrac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20=Ut\cos\alpha$ (or use vertical motion equation)	A1 **M1**	1.1b
	$t=\dfrac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 **A1**	1.1b
		(3)	

HOME

5.

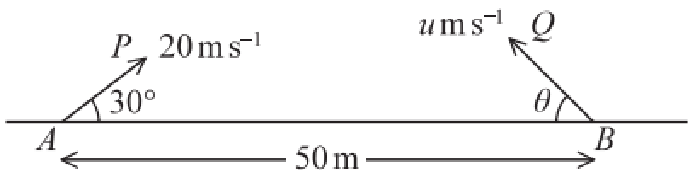


Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed $20\,\text{m s}^{-1}$ at 30° to AB.

Ball Q is projected from B with speed $u\,\text{m s}^{-1}$ at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

A2 2019

Projectiles

(a)	Horizontal speed = 20 cos 30° = $10\sqrt{3}$ m s^{-1}	B1	This mark is given for a correct expression for the horizontal speed of *P*
	$v = u + at$ Vertical speed = 20 sin 30° – 19.6 = –9.6 m s^{-1}	M1	This mark is given for a method to find the vertical speed of *P*
		A1	This mark is given for a correct value for the vertical speed of *P*
	$\theta = \tan^{-1} \pm \frac{9.6}{10\sqrt{3}}$	M1	This mark is given finding an expression for the value of θ
	Speed = $\sqrt{(100 \times 3) + 9.6^2}$	M1	This mark is given for using Pythagoras to find the magnitude of the speed of *P*
	9.8 m s^{-1} downwards at 29.0° to the horizontal	A1	This mark is given for finding the correct velocity of *P* (showing both magnitude and direction)
(b)	Sum of horizontal distances = 50 m	M1	This mark is given for stating the sum of the horizontal distances
	$(u \cos \theta) \times 2 = 50 - (20 \cos 30°) \times 2$ $u \cos \theta = 25 - 20 \cos 30°$	A1	This mark is given for a correct expression for the horizontal distance
	Vertical distances equal $(20 \sin 30°) \times 2 - \frac{g}{2} \times 4 = (u \sin \theta) \times 2 - \frac{g}{2} \times 4$	M1	This mark is given for equating the vertical distances
	$u \sin \theta = 20 \sin 30°$	A1	This mark is given for a correct expression for the vertical distance
	$\theta = 52.5°$, $u = 12.6$ m s^{-1}	M1	This mark is given for a correct method to find θ and u
		A1	This mark is given for finding correct values of θ and u
(c)	For example: The effect of the wind The effect of the spinning of the balls The size of the balls	B1	This mark is given for one correct limitation of the model stated

5.

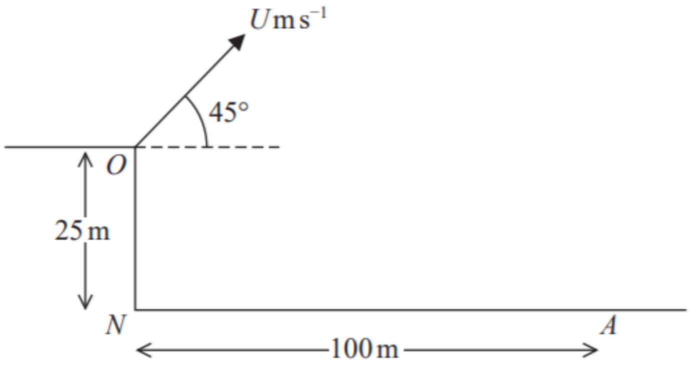


Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where $AN = 100$ m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

Projectiles

5(a)	Using horizontal motion	M1	3.3
	$U\cos 45^\circ t = 100$	A1	1.1b
	Using vertical motion	M1	3.4
	$U\sin 45^\circ t - \frac{1}{2}gt^2 = -25$	A1	1.1b
	Solve problem by eliminating t and solving for U	M1	3.1b
	$U = 28$*	A1*	1.1b
		(6)	
5(b)	Using vertical motion	M1	3.4
	$0^2 = (28\sin 45^\circ)^2 - 2gh$	A1	1.1b
	Greatest height = 45 m	A1	1.1b
		(3)	
5(c)	New value > 28	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of g, spin of ball, include size of the ball, not model as a particle, shape of ball	B1	3.5c
		(1)	
			(11 marks)

A small stone is projected with speed $65\,\text{m}\,\text{s}^{-1}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

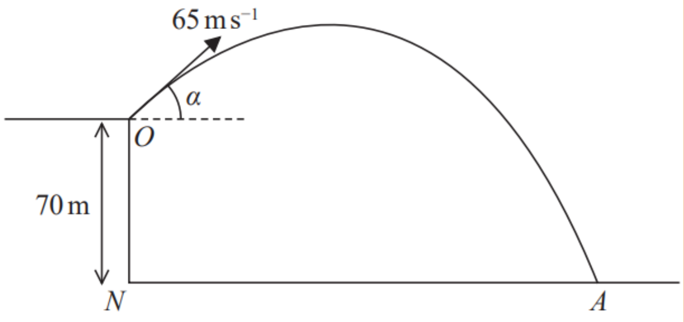


Figure 3

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m}\,\text{s}^{-2}$

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

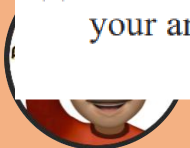
(b) find the speed of the stone at the instant just before it hits the ground at A.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)



Question	Scheme	Marks	AOs
	Note that $g = 10$; penalise once for whole question if $g = 9.8$		
4(a)	Use $s = ut + \frac{1}{2}at^2$ vertically or any complete method to give an equation in t only	M1	3.4
	$-70 = 65\sin\alpha \times t - \frac{1}{2} \times g \times t^2$	A1	1.1b
		M (A)1	1.1b
	$t = 7$ (s)	A1	1.1b
		(4)	
4(b)	Horizontal velocity component at A = $65\cos\alpha$ (60)	B1	3.4
	Complete method to find vertical velocity component at A	M1	3.4
	$65\sin\alpha - g \times 7$ **OR** $\sqrt{(-25)^2 + 2g \times 70}$ (45)	A1**ft**	1.1b
	Sub for trig and square, add and square root : $\sqrt{60^2 + (-45)^2}$	M1	3.1b
	75 Accept 80 (m s^{-1})	A1	1.1b
		(5)	
4(c)	e.g. an approximate value of g has been used, the dimensions of the stone could affect its motion, spin of the stone, $g = 10$ instead of 9.8 has been used, g has been assumed to be constant, wind effect, shape of the stone	B1	3.5b
		(1)	
		(10 marks)	

A2 2022

Projectiles

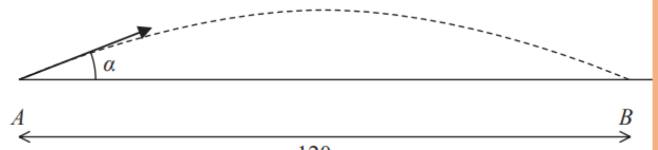


Figure 3

A golf ball is at rest at the point A on horizontal ground.

The ball is hit and initially moves at an angle α to the ground.

The ball first hits the ground at the point B, where $AB = 120\,\text{m}$, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is $U\,\text{m s}^{-1}$

Using this model,

(a) show that $U^2 \sin\alpha \cos\alpha = 588$

(6)

The ball reaches a maximum height of 10 m above the ground.

(b) Show that $U^2 = 1960$

(4)

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from A to B, is now modelled as that of a particle whose initial speed is $V\,\text{m s}^{-1}$

This refined model is used to calculate a value for V

(c) State which is greater, U or V, giving a reason for your answer.

(1)

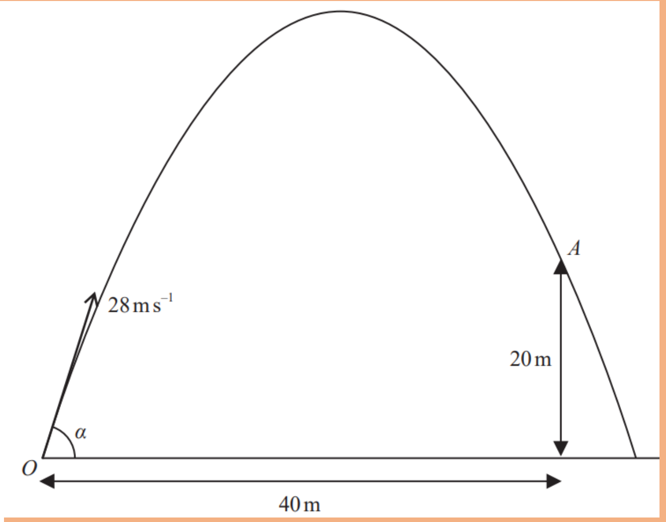
(d) State one further refinement to the model that would make the model more realistic.

(1)

A small ball is projected with speed $28\,\text{m s}^{-1}$ from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.



The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

(3)

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

A2 2023

Projectiles

	N.B. In this question, allow misread of α for a.	
5(a)	Use horizontal motion to give an equation in T and α only: $28\cos\alpha \times T = 40$	M1
	$T = \dfrac{10}{7\cos\alpha}$ *	A1*
		(2)
5(b)	Use vertical motion to give an equation in T and α only	M1
	$20 = (28\sin\alpha)T - \frac{1}{2}gT^2$	A1
	Eliminate T to give an unsimplified equation in α only: $20 = (28\sin\alpha) \times \dfrac{10}{7\cos\alpha} - \frac{1}{2}g\left(\dfrac{10}{7\cos\alpha}\right)^2$	M1
	Use $\sec^2\alpha = 1 + \tan^2\alpha$ oe to give an unsimplified equation in **tan** α **only** : $20 = 40\tan\alpha - \frac{1}{2}g \times \frac{100}{49}(1 + \tan^2\alpha)$	M1
	$\tan^2\alpha - 4\tan\alpha + 3 = 0$ * (allow $0 = \tan^2\alpha - 4\tan\alpha + 3$)	A1*
		(5)
5(c)	Solve and use of $\tan\alpha = 3$ or $\sin\alpha = \dfrac{3}{\sqrt{10}}$ or $\alpha = 71.565..^\circ$ to find an equation in H only.	M1
	$0 = (28\sin\alpha)^2 - 2gH$ **where** $\tan\alpha = 3$ $(\alpha = 71.565..^\circ)$	M1
	H = 36 or 36.0 (m)	A1
		(3)
5(d)	e.g. spin of the ball, the wind, the dimensions or shape of the ball, ball is modelled as a particle, uses an inaccurate value of g, motion takes place in 3D not in 2D, g could be variable. B0 if mass or weight are mentioned. B0 for ground may not be horizontal.	B1
		(1)

A2 2024

Projectiles

At time $t = 0$, a small stone is projected with velocity $35\,\text{m s}^{-1}$ from a point O on horizontal ground.

The stone is projected at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

In an initial model

- the stone is modelled as a particle P moving freely under gravity
- the stone hits the ground at the point A

Figure 4 shows the path of P from O to A.

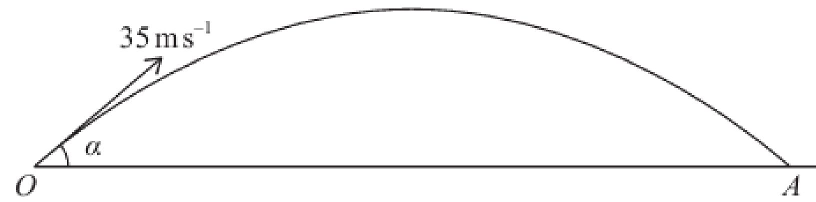


Figure 4

For the motion of P from O to A

- at time t seconds, the horizontal distance of P from O is x metres
- at time t seconds, the vertical distance of P above the ground is y metres

(a) Using the model, show that

$$y = \frac{3}{4}x - \frac{1}{160}x^2$$

(6)

(b) Use the answer to (a), or otherwise, to find the length OA.

(2)

Using the model, the greatest height of the stone above the ground is found to be H metres.

(c) Use the answer to (a), or otherwise, to find the value of H.

(2)

- The model is refined to include air resistance.

Using this new model, the greatest height of the stone above the ground is found to be K metres.

(d) State which is greater, H or K, justifying your answer.

(1)

(e) State one limitation of this refined model.

(1)

HOME

5(a)	Using horizontal motion, $s = ut$, with 35 resolved	M1
	$x = 35\cos\alpha \times t$	A1
	Using vertical motion, $s = ut + \frac{1}{2}at^2$, with 35 resolved	M1
	$y = 35\sin\alpha \times t - \frac{1}{2}gt^2$	A1
	Eliminate t: $y = 35\sin\alpha \times \frac{x}{35\cos\alpha} - \frac{1}{2}g\left(\frac{x}{35\cos\alpha}\right)^2$	DM1
	$y = \frac{3}{4}x - \frac{1}{160}x^2$ *	A1*
	N.B. No marks available if they just quote the equation of the path.	
	ALTERNATIVE: they do (b) and/or (c) first <u>using a suvat method</u> Assume $y = ax^2 + bx + c$ Use any three of (0,0), (120,0) from part (b), (60,22.5) from part (c) or $\frac{dy}{dx} = \frac{3}{4}$ at $x = 0$ to find a, b, and c. **M1A1, M1A1, DM1A1*** for finding each of a, b and c and stating final answer in correct form. **N.B.** If they realise that $c = 0$, and just use $y = ax^2 + bx$, that could score M1A1. Enter marks on ePEN in the order in which a, b and c are found: e.g. $x = 0, y = 0 \Rightarrow c = 0$ M1A1 $x = 0, \frac{dy}{dx} = 2ax + b = \frac{3}{4} \Rightarrow b = \frac{3}{4}$ M1 A1 $x = 120, y = 0 \Rightarrow 0 = 120^2 a + 120 \times \frac{3}{4} \Rightarrow a = \frac{-1}{160}$ DM1 so, $y = \frac{3}{4}x - \frac{1}{160}x^2$ A1*	
		(6)
5(b)	**ALT 1** $0 = \frac{3}{4}x - \frac{1}{160}x^2$ and solve for x **ALT 2** $\frac{dy}{dx} = \frac{3}{4} - \frac{x}{80} = 0 \Rightarrow x = 60$ **and** $OA = 2 \times 60$ **ALT 3**	M1

	A complete *suvat* method to find OA: e.g. $0 = 35\sin\alpha \times t - \frac{1}{2}gt^2$ or $0 = 35\sin\alpha - g\frac{t}{2}$ or $-35\sin\alpha = 35\sin\alpha - gt$ to find t $\left(= \frac{70\sin\alpha}{g} = \frac{30}{7}\right)$ **AND** $(OA =) 35\cos\alpha \times t = 35\cos\alpha \times \frac{70\sin\alpha}{g}$ **N.B. OR** use the calculator to input the equation of the path which then gives $y_{max} = 45/2$ when $x = 60$ **with no working**. so $OA = 2 \times 60$		
	$(OA =)$ 120 (m)	A1	1.1b
		(2)	
5(c)	**ALT 1** $H = \frac{3}{4} \times 60 - \frac{1}{160} \times 60^2$ **ALT 2** $y = \frac{-1}{160}(x^2 - 120x) = 22.5 - \frac{1}{160}(x - 60)^2$ so max $y = 45/2$ or 22.5 **ALT 3** $\frac{dy}{dx} = \frac{3}{4} - \frac{2x}{160} = 0 \Rightarrow x = 60$ then find y when $x = 60$ **ALT 4** A complete *suvat* method: e.g. $H = \frac{(35\sin\alpha)^2}{2g}$ **or** $0 = 35\sin\alpha - gt$ to find the time to top, $t = \frac{35\sin\alpha}{g}$, or use half their time they found in (b) **AND** $H = 35\sin\alpha \times \frac{35\sin\alpha}{g} - \frac{1}{2}g\left(\frac{35\sin\alpha}{g}\right)^2$ or $\left(\frac{35\sin\alpha + 0}{2}\right) \times \frac{35\sin\alpha}{g}$ **N.B. OR** use the calculator to input the equation of the path which then gives $y_{max} = 45/2$ (when $x = 60$) with no working.	M1	3.1b
	$(H =)$ 22.5 Accept 23	A1	1.1b
5(d)	H is greater (or K is smaller), as air resistance would slow the particle down oe.	B1	3.5a
		(1)	
5(e)	e.g. the inaccuracy of using 9.8 m s^{-2} for g	B1	3.5b
		(1)	
			(12 marks)

HOME

Variable Acceleration (including A2 functions and vectors)

6. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

Question	Scheme	Marks	AOs
6	Integrate **a** w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of **C**)	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1ft	1.1b
			(6 marks)

6. At time t seconds, where $t \geqslant 0$, a particle P moves in the x-y plane in such a way that its velocity $\mathbf{v}\ \mathrm{m\,s^{-1}}$ is given by

$$\mathbf{v} = t^{-\frac{1}{2}}\mathbf{i} - 4t\mathbf{j}$$

When $t = 1$, P is at the point A and when $t = 4$, P is at the point B.

Find the exact distance AB.

(6)

A2 2018

Variable Acceleration A2

Question	Scheme	Marks	AOs
6.	Integrate **v** w.r.t. time	M1	1.1a
	$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j}\ \ (+\ \mathbf{C})$	A1	1.1b
	Substitute $t = 4$ and $t = 1$ into their **r**	M1	1.1b
	$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C});\ t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4, -32); (2, -2)$	A1	1.1b
	$\sqrt{2^2 + (-30)^2}$	M1	1.1b
	$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
		(6)	
		(6 marks)	

HOME

1. [*In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geqslant 0$, a particle, P, moves so that its velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})\,\mathrm{m}$.

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

A2 2019

Variable Acceleration A2

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\mathbf{a} = 6\mathbf{i} - \frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	M1	This mark is given for a method to differentiate the expression for **v**
		A1	This mark is given for correctly differentiating the expression for **v**
	$= 6\mathbf{i} - 15\mathbf{j}$ m s^{-1}	A1	This mark is given for substituting $t = 4$ to find a correct vector expression for the acceleration of P
(b)	$\mathbf{r} = (\mathbf{r}_0) + 3t^2\mathbf{i} - 2t^{\frac{5}{2}}\mathbf{j}$	M1	This mark is given for a method to integrate the expression for **v**
		A1	This mark is given for correctly integrating the expression for **v**
	$(-20\mathbf{i} + 20\mathbf{j}) + (48\mathbf{i} - 64\mathbf{j})$ $= 28\mathbf{i} - 44\mathbf{j}$ m	A1	This mark is given for substituting $t = 4$ to find a correct position vector of P

3. (i) At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}\,\mathrm{m\,s^{-2}}$ is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $36\mathbf{i}\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P when $t = 4$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

(ii) At time t seconds, where $t \geqslant 0$, a particle Q moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of t at the instant when the speed of Q is $5\,\mathrm{m\,s^{-1}}$

(6)

Variable Acceleration A2

3(i)(a)	Integrate **a** wrt t to obtain velocity	M1	3.4
	$\mathbf{v} = (t - 2t^2)\mathbf{i} + \left(3t - \frac{1}{3}t^3\right)\mathbf{j}$ (+C)	A1	1.1b
	$8\mathbf{i} - \frac{28}{3}\mathbf{j}$ (m s^{-1})	A1	1.1b
		(3)	
3(i)(b)	Equate **i** component of **v** to zero	M1	3.1a
	$t - 2t^2 + 36 = 0$	A1**ft**	1.1b
	$t = 4.5$ (ignore an incorrect second solution)	A1	1.1b
		(3)	
3(ii)	Differentiate **r** wrt to t to obtain velocity	M1	3.4
	$\mathbf{v} = (2t - 1)\mathbf{i} + 3\mathbf{j}$	A1	1.1b
	Use magnitude to give an equation in t only	M1	2.1
	$(2t-1)^2 + 3^2 = 5^2$	A1	1.1b
	Solve problem by solving this equation for t	M1	3.1a
	$t = 2.5$	A1	1.1b
		(6)	
			(12 marks)

5. At time t seconds, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\mathrm{m\,s^{-1}}$

(6)

A2 2021

Variable Acceleration A2

Question	Scheme	Marks	AOs
	Allow column vectors throughout this question		
5(a)	Differentiate **v** wrt t	M1	3.1a
	$\frac{3}{2}t^{-\frac{1}{2}}\mathbf{i} - 2\mathbf{j}$ isw	A1	1.1b
		(2)	
5(b)	$3t^{\frac{1}{2}} = 2t$	M1	2.1
	Solve for t	DM1	1.1b
	$t = \frac{9}{4}$	A1	1.1b
		(3)	
5(c)	Integrate **v** wrt t	M1	3.1a
	$\mathbf{r} = 2t^{\frac{3}{2}}\mathbf{i} - t^2\mathbf{j}(+\mathbf{C})$	A1	1.1b
	$t = 1, \mathbf{r} = -\mathbf{j} \Rightarrow \mathbf{C} = -2\mathbf{i}$ so $\mathbf{r} = 2t^{\frac{3}{2}}\mathbf{i} - t^2\mathbf{j} - 2\mathbf{i}$	A1	2.2a
		(3)	
5(d)	$\sqrt{(3t^{\frac{1}{2}})^2 + (2t)^2} = 10$ or $(3t^{\frac{1}{2}})^2 + (2t)^2 = 10^2$	M1	2.1
	$9t + 4t^2 = 100$	M(A)1	1.1b
	$t = 4$	A1	1.1b
	$\mathbf{r} = 14\mathbf{i} - 16\mathbf{j}$	M1	1.1b
	$\sqrt{14^2 + (-16)^2}$	M1	3.1a
	$\sqrt{452}$ $(2\sqrt{113})$ (m)	A1	1.1b
		(6)	
		(14 marks)	

HOME

1. [*In this question, position vectors are given relative to a fixed origin.*]

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j})$ m.

(c) Find the position vector of P at time $t = 1$ second.

(4)

A2 2022

Variable Acceleration A2

1(a)	Put $t = 2$ in $\mathbf{v}$ and use Pythagoras: $\sqrt{12^2 + (-6\sqrt{2})^2}$	M1	3.1a
	$\sqrt{216}, 6\sqrt{6}$ or 15 or better (m s^{-1})	A1	1.1b
		(2)	
1(b)	Differentiate $\mathbf{v}$ wrt t to obtain $\mathbf{a}$	M1	3.4
	$6t\mathbf{i} - 3t^{-\frac{1}{2}}\mathbf{j}$ oe (m s^{-2}) isw	A1	1.1b
		(2)	
1(c)	Integrate $\mathbf{v}$ wrt t to obtain $\mathbf{r}$	M1	3.4
	$\mathbf{r} = t^3\mathbf{i} - 4t^{\frac{3}{2}}\mathbf{j}$ (+C)	A1	1.1b
	$(\mathbf{i} - 4\mathbf{j}) = 4^3\mathbf{i} - 4 \times 4^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$	M1	3.1a
	$(-62\mathbf{i} + 24\mathbf{j})$ (m) isw e.g. if they go on to find the distance.	A1	1.1b
		(4)	
			(8 marks)

3. At time t seconds, where $t \geqslant 0$, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$ **(3)**

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$ **(2)**

(c) the acceleration of P at time t seconds **(2)**

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$ **(2)**

3(a)	7**i** – 3**j** seen or implied by Pythagoras	B1
	Use Pythagoras: $\sqrt{7^2+(-3)^2}$	M1
	$\sqrt{58}$, 7.6 or better ($\mathrm{m\ s^{-1}}$)	A1
		(3)
3(b)	$t^2-3t+7=2t^2-3$ **OR** $\frac{t^2-3t+7}{2t^2-3}=\frac{1}{1}$ = 1	M1
	t = 2 only	A1
		(2)
3(c)	Differentiate **v** wrt *t* to give a vector.	M1
	$(2t-3)\mathbf{i}+4t\mathbf{j}$	A1
		(2)
3(d)	$2t-3=0$	M1
	t = 1.5	A1
		(2)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

[In this question, **i** *is a unit vector due east and* **j** *is a unit vector due north. Position vectors are given relative to a fixed origin O.]*

At time t seconds, $t \geqslant 1$, the position vector of a particle P is $\mathbf{r}$ metres, where

$$\mathbf{r} = ct^{\frac{1}{2}}\mathbf{i} - \frac{3}{8}t^2\mathbf{j}$$

and c is a constant.

When $t = 4$, the bearing of P from O is 135°

(a) Show that $c = 3$

(3)

(b) Find the speed of P when $t = 4$

(4)

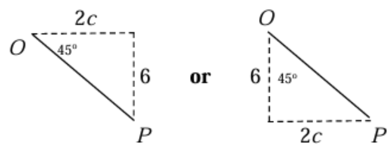
When $t = T$, P is accelerating in the direction of $(-\mathbf{i} - 27\mathbf{j})$.

(c) Find the value of T.

(4)

A2 2024

Variable Acceleration A2

4(a)	**ALTERNATIVES** when $t = 4$ is substituted at the beginning.		
	$2c\mathbf{i} - 6\mathbf{j}$ or as a column vector, seen or implied.	B1	1.1b
	ALT 1  **AND** **either** $\tan 45^o = \frac{2c}{6} \Rightarrow 2c = 6$ **or** **states** isosceles triangle so $2c = 6$ **N.B.** In both of the above,we must see the justification for the equation. **ALT 2** $\tan 135^o = \frac{2c}{-6} \Rightarrow 2c = 6$ **N.B.** M0 if they are using the wrong bearing.	M1	3.1a
	$c = 3$ *	A1*	2.2a

4(b)	Differentiate **r** wrt t to obtain **v**	M1	2.1
	$\mathbf{v} = 3\times\frac{1}{2}t^{-\frac{1}{2}}\mathbf{i} - \frac{3}{8}\times 2t\mathbf{j} = \frac{3}{2}t^{-\frac{1}{2}}\mathbf{i} - \frac{3}{4}t\mathbf{j}$ oe	A1	1.1b
	Put $t = 4$ into **both** components and use Pythagoras: $\sqrt{\left(\frac{3}{4}\right)^2 + (-3)^2}$	M1	3.1a
	$\sqrt{\frac{153}{16}}$ or $\frac{\sqrt{153}}{4}$ or $\frac{3\sqrt{17}}{4}$ or $3\sqrt{\frac{17}{16}} = 3.0923..$ (m s^{-1})	A1	1.1b
		(4)	
4(c)	Differentiate their **v** wrt t to obtain **a**	M1	3.4
	$\mathbf{a} = -\frac{3}{4}t^{-\frac{3}{2}}\mathbf{i} - \frac{3}{4}\mathbf{j}$	A1	1.1b
	$\frac{-\frac{3}{4}T^{-\frac{3}{2}}}{-\frac{3}{4}} = \frac{-1}{-27}$ oe	M1	2.1
	$(T =)\ 9$	A1	1.1b
		(4)	

HOME

Forces (including slopes and friction)

7. A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

Question	Scheme	Marks	AOs
7(a)	$R = mg\cos\alpha$	B1	3.1b
	Resolve parallel to the plane	M1	3.1b
	$-F - mg\sin\alpha = -0.8mg$	A1	1.1b
	$F = \mu R$	M1	1.2
	Produce an equation in μ only and solve for μ	M1	2.2a
	$\mu = \frac{1}{4}$	A1	1.1b
		(6)	
(b)	Compare $\mu mg\cos\alpha$ with $mg\sin\alpha$	M1	3.1b
	Deduce an appropriate conclusion	A1 ft	2.2a
		(2)	
		(8 marks)	

7.

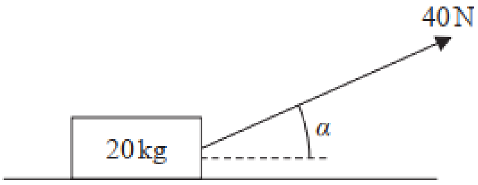


Figure 1

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan\alpha = \frac{3}{4}$

The tension in the handle is 40 N.
The coefficient of friction between the crate and the floor is 0.14
The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate.

(6)

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

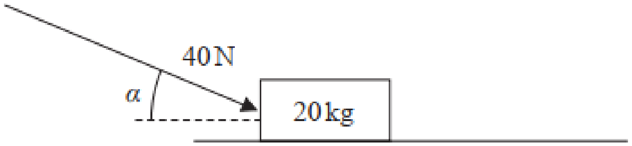


Figure 2

(b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)

A2 2018

Forces A2

Question	Scheme	Marks	AOs
7(a)	Resolve vertically	M1	3.1b
	$R + 40\sin\alpha = 20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40\cos\alpha - F = 20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or 0.40 (m s^{-2})	A1	2.2a
		(6)	
(b)	Pushing will increase R which will increase available F	B1	2.4
	Increasing F will decrease a * GIVEN ANSWER	B1*	2.4
		(2)	
			(8 marks)

HOME

1. A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ

Brick P is in equilibrium and on the point of sliding down the plane.

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick P **(2)**

(b) show that $\mu = \frac{3}{4}$ **(4)**

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$

(c) Explain briefly why brick Q will remain at rest on the plane. **(1)**

Brick Q is now projected with speed $0.5\,\text{m}\,\text{s}^{-1}$ down a line of greatest slope of the plane.

Brick Q is modelled as a particle.

Using the model,

(d) describe the motion of brick Q, giving a reason for your answer. **(2)**

A2 2020

Forces A2

1.(a)	Resolve perpendicular to the plane	M1	3.4
	$R = mg\cos\alpha = \frac{4}{5}mg$	A1	1.1b
		(2)	
1(b)	Resolve parallel to the plane or horizontally or vertically	M1	3.4
	$F = mg\sin\alpha$ or $R\sin\alpha = F\cos\alpha$	A1	1.1b
	Use $F = \mu R$ and solve for μ	M1	2.1
	$\mu = \frac{3}{4}$ *	A1*	2.2a
		(4)	
1(c)	The forces acting on *Q* will still balance as the *m*'s cancel oe Other possibilities: e.g. the friction will increase in the same proportion as the weight component or force down the plane. The force pulling the brick down the plane increases by the same amount as the friction oe This mark can be scored if they do the calculation.	B1	2.4
		(1)	
1(d)	Brick *Q* slides down the plane with **constant** speed.	B1	2.4
	No resultant force down the plane (so no acceleration) oe	B1	2.4
	These marks can be scored if they do the calculation.	**(2)**	
			(9 marks)

HOME

2.

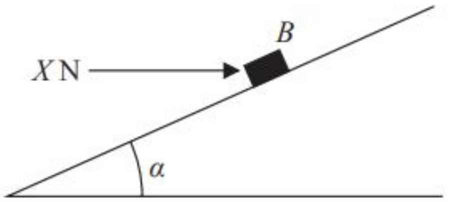


Figure 1

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A small block B of mass 5 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 1.

The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block B is modelled as a particle.

The magnitude of the normal reaction of the plane on B is 68.6 N.

Using the model,

(a) (i) find the magnitude of the frictional force acting on B,

(3)

(ii) state the direction of the frictional force acting on B.

(1)

The horizontal force of magnitude X newtons is now removed and B moves down the plane.

Given that the coefficient of friction between B and the plane is 0.5

(b) find the acceleration of B down the plane.

(6)

A2 2022

Forces A2

2(a)(i)	Resolve vertically	M1
	F acting UP the plane: **OR** *F* acting DOWN the plane: $(\uparrow) F\sin\alpha + 68.6\cos\alpha = 5g$ $\quad -F\sin\alpha + 68.6\cos\alpha = 5g$ Other possible equations from which *X* would need to be eliminated to give an equation in *F* **only** to earn the M mark are shown below. The equation in *F* **only** must then be correct to earn the A mark. Possible equations: $(\nwarrow) 68.6 = X\sin\alpha + 5g\cos\alpha$ (leads to $X = 49$ with $g = 9.8$) *F* acting UP the plane: **OR** *F* acting DOWN the plane: $(\nearrow) F + X\cos\alpha = 5g\sin\alpha$ $\quad -F + X\cos\alpha = 5g\sin\alpha$ $(\rightarrow) F\cos\alpha + X = 68.6\sin\alpha$ $\quad -F\cos\alpha + X = 68.6\sin\alpha$	A
	9.8 (N) (49/5 is A0) **N.B.** If sin and cos are interchanged in all equations, this leads to an answer of 9.8 in the wrong direction and can only score (a) (i)M1A0A0 (ii) A0	A1
		(3)
2(a)(ii)	Down the plane (Allow down or downwards or an arrow ↙, but must appear as the answer to (a) (ii) not just on the diagram.)	A1
		(1)
2(b)	**N.B.** If they use $R = 68.6$ in this part, the maximum they can score is M1A1M0A0M0A0 If they use $F = 9.8$ or their *F* from (a) in this part, the maximum they can score is M1A1M0A0M0A0	
	Equation of motion down the plane	M1
	$5g\sin\alpha - F = 5a$ Allow (-*a*) instead of *a*	A1
	Resolve perpendicular to the plane	M1
	$R = 5g\cos\alpha$	A1
	$F = 0.5R$ seen	M1
	$a = 1.96$ or 2.0 or 2 (m s^{-2}) or $\frac{1}{5}g$	A1
		(6)

HOME

2.

Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at $1.4\,\text{m}\,\text{s}^{-2}$

(b) Find the value of F

(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

Question	Scheme	Marks
2(a)	Resolve vertically, $R = 5g = 49$ (N)	B1
		(1)
2(b)	Equation of motion: $28 - F = 5 \times 1.4$	M1
	$F = 21$	A1
		(2)
2(c)	$\mu = 0.43$ (2sf required)	B1 **ft**
		(1)

1.

Figure 1

Figure 1 shows a particle P of mass 0.5 kg at rest on a rough horizontal plane.

(a) Find the magnitude of the normal reaction of the plane on P.

(1)

The coefficient of friction between P and the plane is $\frac{2}{7}$

A horizontal force of magnitude X newtons is applied to P.

Given that P is now in limiting equilibrium,

(b) find the value of X.

(2)

1(a)	$0.5g$, $\frac{1}{2}g$ or 4.9 (N) seen	B1	3.4
		(1)	
1(b)	$\frac{2}{7} \times 4.9$ oe seen	M1	3.1a
	1.4, 1.40 or $\frac{1}{7}g$	A1	1.1b
		(2)	
			(3 marks)

3.

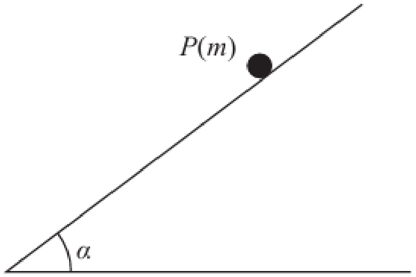


Figure 3

A particle P of mass m is held at rest at a point on a rough inclined plane, as shown in Figure 3.

It is given that

- the plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$
- the coefficient of friction between P and the plane is μ, where $\mu < \frac{5}{12}$

The particle P is released from rest and slides down the plane.
Air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on P,
(2)

(b) show that, as P slides down the plane, the acceleration of P down the plane is

$$\frac{1}{13}g(5 - 12\mu)$$

(4)

(c) State what would happen to P if it is released from rest but $\mu \geqslant \frac{5}{12}$
(1)

3(a)	$(R=)\, mg\cos\alpha$	M1	3.4
	$=\frac{12}{13}mg$	A1	1.1b
		(2)	
3(b)	Equation of motion down the plane	M1	2.1
	$mg\sin\alpha - F = ma$ or $mg\sin\alpha - F = -ma$	A1	1.1b
	$(F=)\ \mu\times$ their R	M1	3.4
	$\frac{1}{13}g(5-12\mu)$ *	A1*	2.2a
		(4)	
3(c)	*P* wouldn't move	B1	2.4
		(1)	
			(7 marks)

BICEN MATHS

HOME

Connected Particles (including slopes and friction)

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle α to the horizontal ground, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane. Block B hangs freely below P, as shown in Figure 1.

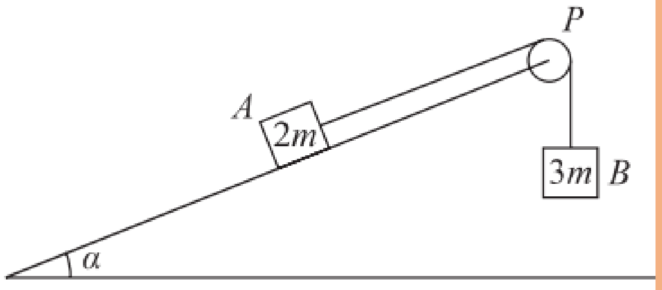


Figure 1

The coefficient of friction between A and the plane is $\frac{2}{3}$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \frac{12mg}{5}$

(8)

After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

(b) Determine whether A will remain at rest, carefully justifying your answer.

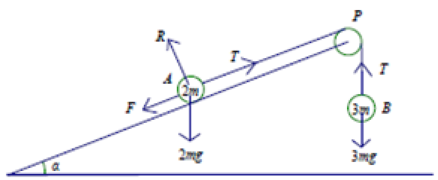
(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)

A2 2019

Connected Particles A2

(a)			
	$R = 2mg\cos\alpha = \frac{24mg}{13}$	B1	This mark is given for using the model to state the normal reaction between A and the plane
	$F_{max} = \frac{2}{3}R = \frac{16mg}{13}$	B1	This mark is given for the use of $F = \mu R$
	Equation of motion for A is $T - F_{max} - 2mg\sin\alpha = 2ma$	M1	This mark is given for a method form an equation of motion for A
		A1	This mark is given for a correct equation of motion for A
	Equation of motion for B is $3mg - T = 3ma$	M1	This mark is given for a method to form an equation of motion for B
		A1	This mark is given for a correct equation of motion for B
	$3mg - \frac{16mg}{13} - \frac{10mg}{13} = 5ma$	M1	This mark is given for a method using the equations of motion for A and B to solve for T
	$T = 3mg - \frac{3mg}{5} = \frac{12mg}{5}$	A1	This mark is given for a full method and correct working to show the answer given
(b)	$F_{max} = \frac{16mg}{13} > \frac{10mg}{13}$ $\frac{10mg}{13}$ is the component of the weight parallel to the slope	M1	This mark is given for a comparison of F_{max} with the component of weight
	Thus A will not move	A1	This mark is given for a fully justified and correct conclusion
(c)	Have the model consider air resistance	B1	This mark is given for one correct refinement stated
	Have the model use an extensible string	B1	This mark is given for one correct refinement stated

HOME

A2 2021

Connected Particles A2

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P, as shown in Figure 1.

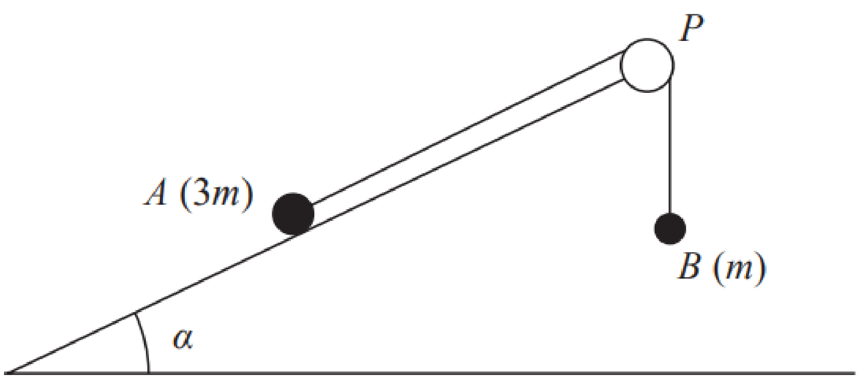


Figure 1

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

(c) sketch a velocity-time graph for the motion of B, from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

(2)

In reality, the string is not light.

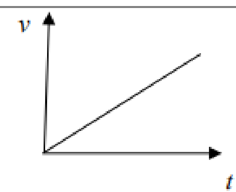
(d) State how this would affect the working in part (b).

(1)

HOME

A2 2021

Connected Particles A2

	Mark parts (a) and (b) together		
2(a)	Equation of motion for A	M1	3.3
	$3mg\sin\alpha - F - T = 3ma$	A1	1.1b
		(2)	
2(b)	Resolve perpendicular to the plane	M1	3.4
	$R = 3mg\cos\alpha$	A1	1.1b
	$F = \frac{1}{6}R$	B1	1.2
	Equation of motion for B **OR** for whole system	M1	3.3
	$T - mg = ma$ **OR** $3mg\sin\alpha - F - mg = 3ma + ma$	A1	1.1b
	Complete method to solve for a	DM1	3.1b
	$a = \frac{1}{10}g$ *	A1*	2.2a
		(7)	
2(c)		B1	1.1b
	e.g. acceleration (of B) is constant; dependent on first B1	DB1	2.4
		(2)	
2(d)	e.g. the tensions in the two equations of motion would be different. Tension on A would be different to tension on B	B1	3.5a
		(1)	
			(12 marks)

HOME

Moments

A2 SAMs

Moments

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

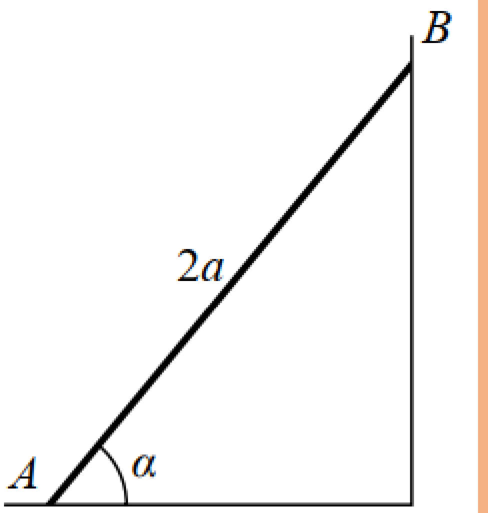


Figure 1

HOME

A2 SAMs

Moments

Question	Scheme	Marks	AOs
9(a)	Take moments about A (or any other complete method to produce an equation in S , W and α only)	M1	3.3
	$Wa\cos\alpha\ +\ 7W2a\cos\alpha\ = S\,2a\sin\alpha$	A1 A1	1.1b 1.1b
	Use of $\tan\alpha\ =\ \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4}R\ (= 2W)$	M1	3.4
	$P_{MAX} = 3W + F$ or $P_{MIN} = 3W - F$	M1	3.4
	$P_{MAX} = 5W$ or $P_{MIN} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

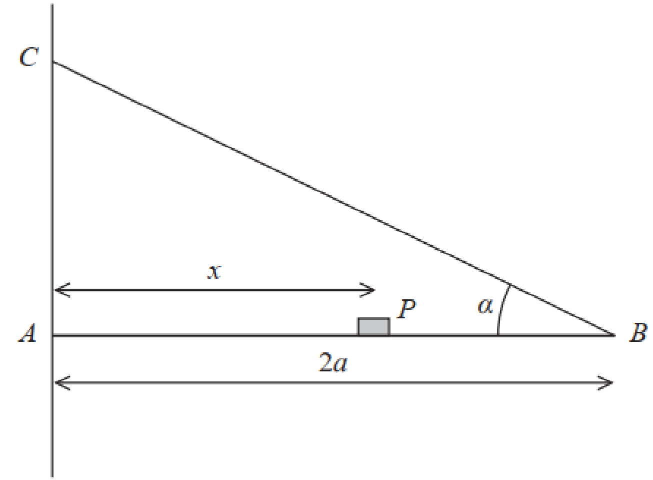


Figure 3

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x + a)}{6a}$
(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.
(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$
(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.
(3)

HOME

A2 2018

Moments

9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \dfrac{Mg(a+3x)}{2a \times \frac{3}{5}} = \dfrac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\dfrac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \dfrac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \dfrac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\dfrac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1ft	1.1b
	$Y = \dfrac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \dfrac{Y}{X}$ or $\dfrac{R\sin\beta}{R\cos\beta}$ $= \dfrac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \dfrac{5}{4}$	A1	2.2a
		(5)	
(d)	$\dfrac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \dfrac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\dfrac{5a}{3}$ from A oe Or just: $x \le \dfrac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 A1	2.4
		(3)	

HOME

A ramp, AB, of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A.

The point of contact between the ramp and the drum is C, where $AC = 5$ m, as shown in Figure 2.

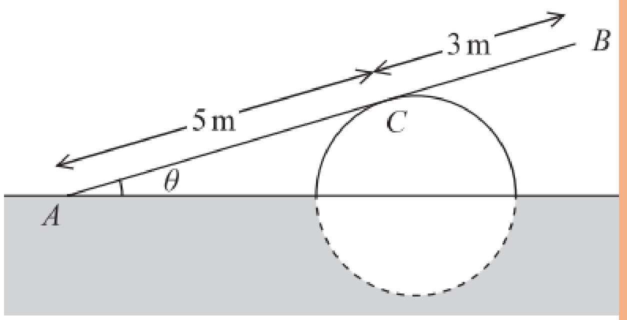


Figure 2

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp.

(1)

(b) Find the magnitude of the resultant force acting on the ramp at A.

(9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

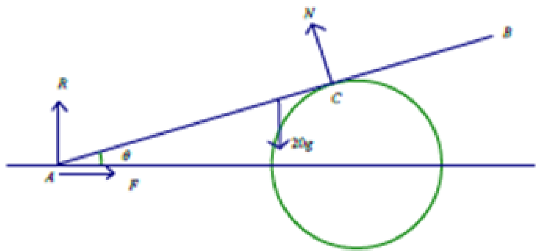
Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C.

(1)

A2 2019

Moments

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The drum is smooth so there is no friction; thus there is no component parallel to the ramp and therefore the reaction is perpendicular to the ramp	B1	This mark is given for a correct explanation stated
(b)			
	M(A): $5N = 20g \times 4\cos\theta$	M1	This mark is given for a method to find moments about A
	$N = 16g\cos\theta$ $N = 150$	A1	This mark is given for a correct value for N
	$\updownarrow$ $R + N\cos\theta = 20g$	M1	This mark is given for finding an equation in R by resolving vertically
	$R + N \times \dfrac{24}{25} = 20g$	A1	This mark is given for a correct equation in R
	$\updownarrow$ $F = N\sin\theta = 20g$	M1	This mark is given for finding an equation in F by resolving vertically
	$F = N \times \dfrac{7}{25}$	A1	This mark is given for a correct equation for F
	$R = 51.5$ N, $F = 42.1$ N	M1	This mark is given for using trigonometry to correctly solve for R and F
	$\lvert\text{Force}\rvert = \sqrt{51.5^2 + 42.1^2} = 66.5$ N	M1	This mark is given for a method to find the resultant force
		A1	This mark is given for correctly finding the resultant force
(c)	The magnitude of the normal reaction will decrease	B1	This mark is given for a correct reason given

A2 2020

Moments

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C.

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ.

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$

(3)

(b) Hence, or otherwise, find the value of μ.

(7)

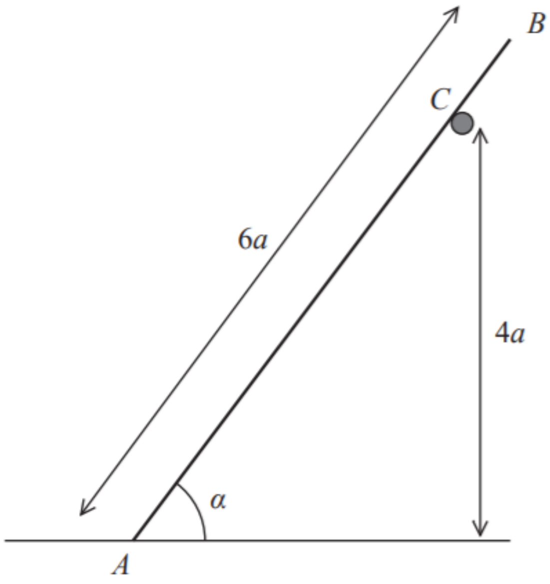


Figure 1

A2 2020

Moments

4(a)	Take moments about A	M1	3.3
	$N \times \frac{4a}{\sin\alpha} = Mg \times 3a\cos\alpha$	A1	1.1b
	$\frac{9Mg}{25}$ *	A1*	1.1b
		(3)	
4(b)	Resolve horizontally	M1	3.4
	$(\rightarrow)\ F = \frac{9Mg}{25}\sin\alpha$	A1	1.1b
	Resolve vertically	M1	3.4
	$(\uparrow)\ R + \frac{9Mg}{25}\cos\alpha = Mg$	A1	1.1b
	Other possible equations: $(\nwarrow), R\cos\alpha + \frac{9Mg}{25} = Mg\cos\alpha + F\sin\alpha$ $(\nearrow), Mg\sin\alpha = F\cos\alpha + R\sin\alpha$ $M(C), Mg.2a\cos\alpha + F.5a\sin\alpha = R.5a\cos\alpha$ $M(G), \frac{9Mg}{25}.2a + F.3a\sin\alpha = R.3a\cos\alpha$ $M(B), Mg.3a\cos\alpha + F.6a\sin\alpha = R.6a\cos\alpha + \frac{9Mg}{25}a$ $(F = \frac{36Mg}{125},\ R = \frac{98Mg}{125})$		
	$F = \mu R$ used	M1	3.4
	Eliminate R and F and solve for μ	M1	3.1b

Alternative equations if they have at A:
X horizontally and Y perpendicular to the rod.

$(\nwarrow), Y + \frac{9Mg}{25} = Mg\cos\alpha + X\sin\alpha$

$(\nearrow), Mg\sin\alpha = X\cos\alpha$

$(\uparrow), \frac{9Mg}{25}\cos\alpha + Y\cos\alpha = Mg$

$(\rightarrow), Y\sin\alpha + \frac{9Mg}{25}\sin\alpha = X$

$M(C), Mg.2a\cos\alpha + X.5a\sin\alpha = Y.5a$ $M(G), \frac{9Mg}{25}.2a + X.3a\sin\alpha = Y.3a$ M1A1 M1A1 $M(B), Mg.3a\cos\alpha + X.6a\sin\alpha = Y.6a + \frac{9Mg}{25}a$ $(X = \frac{4Mg}{3}, Y = \frac{98Mg}{75})$ Then $F = \mu R$ becomes: $X - Y\sin\alpha = \mu Y\cos\alpha$ M1 Eliminate X and Y and solve for μ M1	
$\mu = \frac{18}{49}$ (0.3673…..accept 0.37 or better)	A1
	(7)

HOME

A2 2021

Moments

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

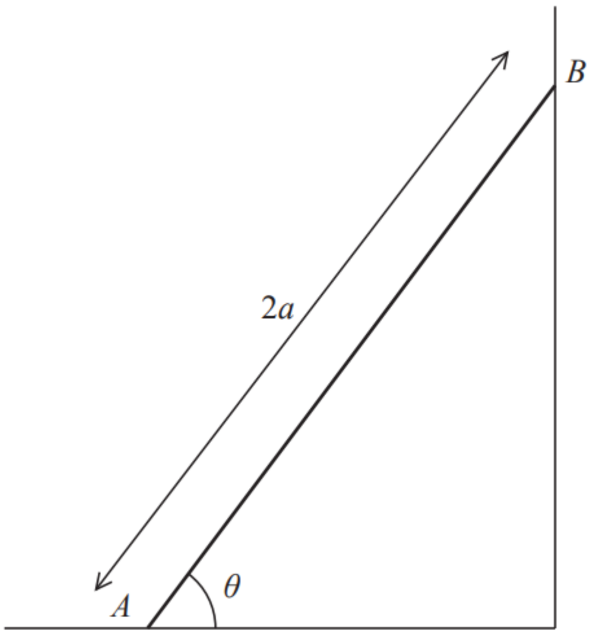


Figure 2

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k.

(5)

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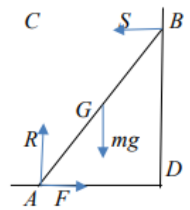
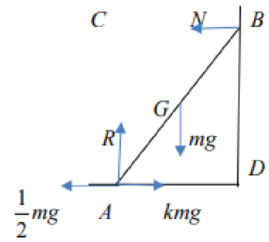
A2 2021

Moments

	Part (a) is a 'Show that..' so equations need to be given in full to earn A marks		
3(a)			
	Moments equation: (M1A0 for a moments inequality)	M1	3.3
	M(A), $mga\cos\theta = 2Sa\sin\theta$ M(B), $mga\cos\theta + 2Fa\sin\theta = 2Ra\cos\theta$ M(C), $F \times 2a\sin\theta = mga\cos\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + 2Sa\sin\theta$ M(G), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$.	A1	1.1b
	($\updownarrow$) $R = mg$ **OR** ($\leftrightarrow$) $F = S$	B1	3.4
	Use their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ **only** (allow DM1 for use of $F = \mu R$ to give an *equation* in μ and θ only)	DM1	2.1
	$\mu \ge \frac{1}{2}\cot\theta$ *	A1*	2.2a
		(5)	

3(b)			
	Moments equation:	M1	3.4
	M(A), $mga\cos\theta = 2Na\sin\theta$ M(B), $mga\cos\theta + 2kmga\sin\theta = 2Ra\cos\theta + \frac{1}{2}mg2a\sin\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + N2a\sin\theta$ M(G), $kmga\sin\theta + Na\sin\theta = \frac{1}{2}mga\sin\theta + Ra\cos\theta$	A1	1.1b
	S.C. M(C), $mga\cos\theta + \frac{1}{2}mg2a\sin\theta = kmg2a\sin\theta$ **M1A1B1** $1 + \frac{5}{4} = \frac{5k}{2}$ **M1** $k = 0.9$ **A1**		
	$N = kmg - F$ **OR** $R = mg$	B1	3.3
	Use their equations (they must have enough) to solve for k (numerical)	DM1	3.1b
	$k = 0.9$ oe	A1	1.1b
		(5)	
			(10 marks)

HOME

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C, where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

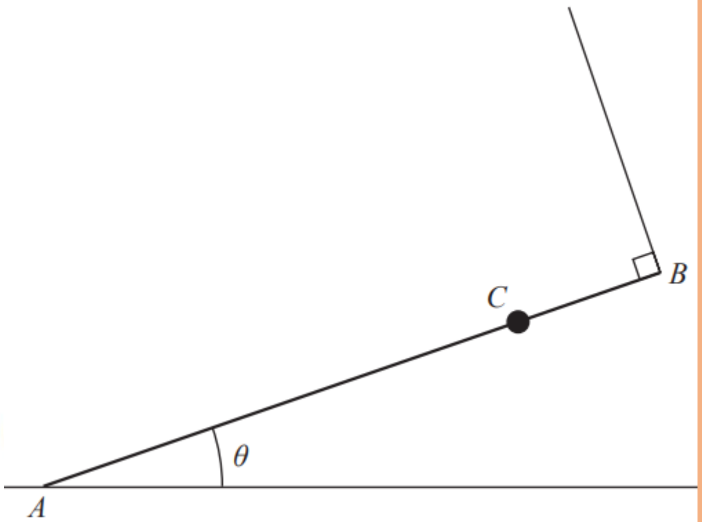


Figure 2

(a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(1)

The tension in the string is T

(b) Show that $T = 2Mg\cos\theta$

(3)

Given that $\cos\theta = \frac{3}{5}$

(c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\frac{57Mg}{25}$

(3)

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

(d) show that $\mu = \frac{8}{19}$

(4)

Moments

4(a)	The horizontal component of T acts to the left and since the **only** other horizontal force is friction, it must act to the right oe	B1
		(1)
4(b)	Take moments about A or any other complete method to obtain **an equation in T, M and θ only.** (see possible equations below that they may use)	M1
	$T.2a = Mga\cos\theta + 2Mg \times 1.5a\cos\theta$ (A0 if a's missing)	A1
	Other possible equations but F and R would need to be eliminated. $(\nwarrow)$, $R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow)$, $R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow)$, $F = T\sin\theta$ M(B), $R.2a\cos\theta = Mga\cos\theta + 2Mg \times 0.5a\cos\theta + F.2a\sin\theta$ M(G), $Fa\sin\theta + Ta = Ra\cos\theta + 2Mg \times 0.5a\cos\theta$ M(C), $R \times 1.5a\cos\theta = T \times 0.5a + Mg \times 0.5a\cos\theta + F \times 1.5a\sin\theta$	
	$T = 2Mg\cos\theta$ *	A1*
		(3)
4(c)	e.g. Resolve vertically	M1
	$(\uparrow)$, $R + T\cos\theta = Mg + 2Mg$	A1
	$R = \dfrac{57Mg}{25}$ *	A1*
		(3)
4(d)	Find an equation containing F e.g. Resolve horizontally	M1
	$(\rightarrow)$, $F = T\sin\theta$	A1
	$F = \mu R$ used i.e. both F and R are substituted.	M1
	$\mu = \dfrac{8}{19}$ *	A1*
		(4)

A2 2023

Moments

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

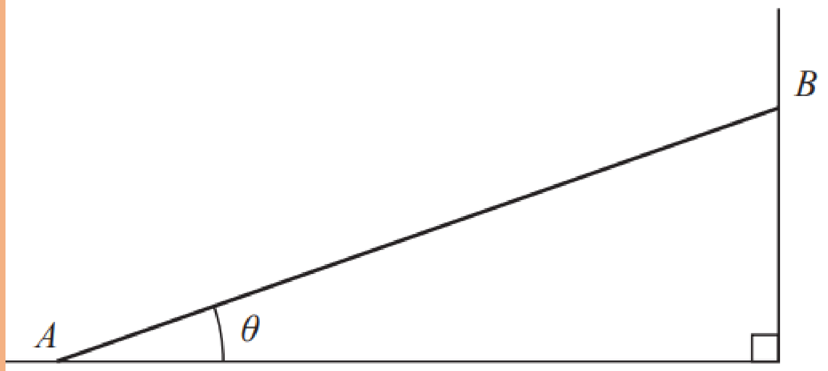


Figure 3

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

HOME

A2 2023

Moments

6(a)	The normal reaction at *B* is acting to the left so it must act to the right, right as it needs to balance (oppose, counter) the force at *B*, right as it prevents the rod from sliding (slipping, falling), right as the weight (mass) of the rod will mean the rod tends to slip left, mass or weight will be pushing the rod to the left so friction will oppose that. **N.B.** You may see an arrow on the diagram at *A*, instead of 'right'. B0 if they say the rod is moving oe Accept towards the wall instead of to the right.	B1
		(1)
6(b)	Take moments about *A*	M1
	$S \times 2a\sin\theta = Mga\cos\theta$	A1
	$S = \frac{1}{2}Mg\cot\theta$ *	A1*
		(3)
6(c)	Resolve vertically, $R = Mg$	B1
	Resolve horizontally, $F = S$	B1
	Other possible equations: Resolve along the rod, $F\cos\theta + R\sin\theta = S\cos\theta + Mg\sin\theta$ Resolve perp to the rod, $R\cos\theta + S\sin\theta = F\sin\theta + Mg\cos\theta$ M(*B*), $R \times 2a\cos\theta = F \times 2a\sin\theta + Mga\cos\theta$ M(*G*), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$ **N.B.** When entering these two B marks on ePEN, **First** B1 is for a **vertical** resolution, **second** B1 is for a **horizontal** resolution, and if either is replaced by a different equation, enter appropriately. If both are replaced by other equations, enter in the order in which they appear in their working.	
	$F = \mu R$	B1
	$\frac{1}{2}Mg \times \frac{4}{3} = \mu Mg$	**dM1**
	$\mu = \frac{2}{3}$ oe Accept 0.67 or better	A1
	S.C. For F „ μR, B0 $\frac{1}{2}Mg \times \frac{4}{3}$ „ μMg M1	

	$\frac{2}{3}$ „ μ A0 **N.B.** If $\mu = \frac{2}{3}$ follows this, they could score all the marks.	
		(5)
6(d)	$\sqrt{F^2 + R^2}$	M1
	$\sqrt{\left(\frac{2}{3}Mg\right)^2 + (Mg)^2}$	M1
	$\frac{1}{3}Mg\sqrt{13}$ or 1.2*Mg* or better	A1
		(3)
6(e)	New value of *S* would be **larger** as the **moment** of the **weight** about ***A*** would be larger	B1
		(1)

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6.

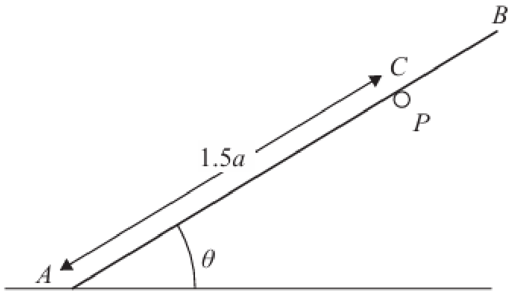


Figure 5

Figure 5 shows a uniform rod AB of mass M and length $2a$.

- the rod has its end A on rough horizontal ground
- the rod rests in equilibrium against a small smooth fixed horizontal peg P
- the point C on the rod, where $AC = 1.5a$, is the point of contact between the rod and the peg
- the rod is at an angle θ to the ground, where $\tan\theta = \frac{4}{3}$

The rod lies in a vertical plane perpendicular to the peg.

The magnitude of the normal reaction of the peg on the rod at C is S.

(a) Show that $S = \frac{2}{5}Mg$

(3)

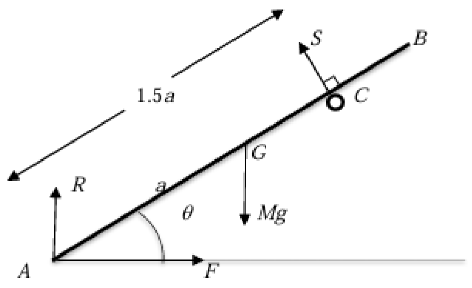
The coefficient of friction between the rod and the ground is μ.

Given that the rod is in limiting equilibrium,

(b) find the value of μ.

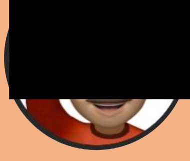
(6)

Moments

6.			
6(a)	Take moments about A	M1	3.1a
	$S\times 1.5a = Mga\cos\theta = (Mga\times\frac{3}{5})$	A1	1.1b
	$S=\frac{2}{5}Mg$ *	A1*	2.2a
		(3)	
6(b)	**N.B. Marks for the equations should be awarded in the order in which they appear on the script.**		
	Resolve horizontally:	M1	3.4
	$F=S\sin\theta$	A1	1.1b
	Resolve vertically:	M1	3.3
	$R=Mg-S\cos\theta$	A1	1.1b
	Other possible equations: (any of which is worth max M1A1) (parallel to the rod): $F\cos\theta+R\sin\theta=Mg\sin\theta$ (perp to the rod): $F\sin\theta+Mg\cos\theta=S+R\cos\theta$ M(B): $(S\times 0.5a)+(R\times 2a\cos\theta)=(Mg\times a\cos\theta)+(F\times 2a\sin\theta)$ M(C): $(R\times 1.5a\cos\theta)=(Mg\times 0.5a\cos\theta)+(F\times 1.5a\sin\theta)$ M(G): $(R\times a\cos\theta)=(S\times 0.5a)+(F\times a\sin\theta)$ **N.B. If they have more than two equations, mark only those that they use to try to find μ**		
	$F=\mu R$ and two of their equations used to solve for μ	DM1	3.1a
	$\mu=\frac{8}{19}=0.42105\ldots$	A1	2.2a

HOME

Constant Acceleration with Vectors



8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(b) Find **r** in terms of t.

(2)

(c) Find the value of t when the boat is north-east of O.

(3)

(d) Find the value of t when the boat is moving in a north-east direction.

(3)

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2} Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}\,t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})\,t^2$	A1	1.1b
		(2)	
(c)	Equating the **i** and **j** components of **r**	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7\,t^2 = 0.6\,t - \frac{1}{2} \leftarrow 0.1\,t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})\,t$	M1	3.1b
	Equating the **i** and **j** components of **v**	M1	3.1b
	$t = 0.75$	A1 ft	1.1b
		(3)	
		(10 marks)	

HOME

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O.*]

A particle P moves with constant acceleration.
At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$
At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})\,\text{m}$.

(a) Show that the magnitude of the acceleration of P is $2.5\,\text{m s}^{-2}$

(4)

At the instant when P leaves the point A, the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})\,\text{m s}^{-2}$

At the instant when P reaches the point B, the direction of motion of P is north east.

(b) Find the time it takes for P to travel from A to B.

(4)

A2 2018

Constant accn. with vectors

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $(7\mathbf{i} - 10\mathbf{j}) = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}\mathbf{a}2^2$	M1	3.1b
	$\mathbf{a} = (1.5\mathbf{i} - 2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{a}\rvert = \sqrt{1.5^2 + (-2)^2}$	M1	1.1b
	$= 2.5$ m s^{-2} * GIVEN ANSWER	A1*	2.1
		(4)	
(b)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 3\mathbf{j}) + 2(1.5\mathbf{i} - 2\mathbf{j})$	M1	3.1b
	$= (5\mathbf{i} - 7\mathbf{j})$	A1	1.1b
	$\mathbf{v} = (5\mathbf{i} - 7\mathbf{j}) + t(4\mathbf{i} + 8.8\mathbf{j}) = (5 + 4t)\mathbf{i} + (8.8t - 7)\mathbf{j}$ and $(5 + 4t) = (8.8t - 7)$	M1	3.1b
	$t = 2.5$ (s)	A1	1.1b
		(4)	
		(8 marks)	

HOME

2. A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T.

(4)

At time $t = 4$ seconds, P is at the point B.

(b) Find the distance AB.

(4)

A2 2019

Constant accn. with vectors

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$ $\mathbf{v} = (-\mathbf{i} + 4\mathbf{j}) + (2\mathbf{i} - 3\mathbf{j})t$	M1	This mark is given for a method to find a vector expression for **v**
	$= (-1 + 2t)\mathbf{i} + (4 - 3t)\mathbf{j}$	A1	This mark is given for finding a correct vector expression for **v**
	$\frac{4-3T}{1+2T} = \frac{-4}{3}$	M1	This mark is given for a correct use of ratios as a method to find the value of T
	$12 - 9T = 4 - 8T$ $T = 12 - 4 = 8$	A1	This mark is given for finding the correct value of T
(b)	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ $\mathbf{s} = (-\mathbf{i} + 4\mathbf{j})t + \frac{1}{2}(2\mathbf{i} - 3\mathbf{j})t^2$	M1	This mark is given for a method to find a vector expression for the distance AB
	$= (-t + t^2)\mathbf{i} + \left(4t - \frac{3}{2}t^2\right)\mathbf{j}$	A1	This mark is given for finding a correct vector expression for the distance AB
	$AB = \sqrt{12^2 + 8^2}$	M1	This mark si given for a method to find the distance AB using Pythagoras and substituting $t = 4$
	$= 14.4$ m	A1	This mark is given for find a correct value for the distance AB

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A2 2020

Constant accn. with vectors

2. A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

A2 2020

Constant accn. with vectors

2(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or integrate to give: $\mathbf{v} = (-2\mathbf{i} + 2\mathbf{j}) + 2(4\mathbf{i} - 5\mathbf{j})$	M1	3.1a
	$(6\mathbf{i} - 8\mathbf{j})$ (m s^{-1})	A1	1.1b
		(2)	
2(b)	Solve problem through use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or integration (M0 if $\mathbf{u} = \mathbf{0}$) **Or** any other complete method e.g use $\mathbf{v} = \mathbf{u} + \mathbf{a}T$ and $\mathbf{r} = \frac{(\mathbf{u}+\mathbf{v})T}{2}$:	M1	3.1a
	$-4.5\mathbf{j} = 2t\mathbf{j} - \frac{1}{2}t^2 5\mathbf{j}$ (**j** terms only)	A1	1.1b
	The first two marks could be implied if they go straight to an algebraic equation.		
	Attempt to equate **j** components to give equation in T only ($-4.5 = 2T - \frac{5}{2}T^2$)	M1	2.1
	$T = 1.8$	A1	1.1b
		(4)	
2(c)	Solve problem by substituting <u>their</u> T value (M0 if $T < 0$) into the **i** component equation to give an equation in λ only: $\lambda = -2T + \frac{1}{2}T^2 \times 4$	M1	3.1a
	$\lambda = 2.9$ or 2.88 or $\frac{72}{25}$ oe	A1	1.1b
		(2)	
Notes: Accept column vectors throughout			**(8 marks)**

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A2 2021

Constant accn. with vectors

1. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\text{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

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Question	Scheme	Marks	AOs
1(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 2$: $\mathbf{v} = 4\mathbf{i} + 2(2\mathbf{i} - 3\mathbf{j})$ **OR integration**: $\mathbf{v} = (2\mathbf{i} - 3\mathbf{j})t + 4\mathbf{i}$, with $t = 2$	M1	3.1a
	$\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$	A1	1.1b
		(2)	
1(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ at $t = 3$: $(\mathbf{i} + \mathbf{j}) + \left[3 \times 4\mathbf{i} + \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR**: find $\mathbf{v}$ at $t = 3$: $4\mathbf{i} + 3(2\mathbf{i} - 3\mathbf{j}) = (10\mathbf{i} - 9\mathbf{j})$ then use $\mathbf{r} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ $(\mathbf{i} + \mathbf{j}) + \left[\frac{1}{2}[4\mathbf{i} + (10\mathbf{i} - 9\mathbf{j})] \times 3\right]$ or $\mathbf{r} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $(\mathbf{i} + \mathbf{j}) + \left[3 \times (10\mathbf{i} - 9\mathbf{j}) - \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR integration:** $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \left[(2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2 + 4t\mathbf{i}\right]$, with $t = 3$	M1	3.1a
	$\mathbf{r} = 22\mathbf{i} - 12.5\mathbf{j}$	A1	2.2a
		(2)	
		(4 marks)	

3. [*In this question*, **i** *and* **j** *are horizontal unit vectors*.]

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})$ N and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})$ N, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0$$

(4)

At time $t = 4$ seconds, P passes through the point B.

Given that $\lambda = 2$

(b) find the length of AB.

(5)

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Constant accn. with vectors

Question	Scheme	Marks
3(a)	$(4\mathbf{i}-\mathbf{j})+(\lambda\mathbf{i}+\mu\mathbf{j}) = (4+\lambda)\mathbf{i}+(-1+\mu)\mathbf{j}$	M1
	Use **ratios** to obtain an equation in λ and μ *only*	M1
	$\frac{(4+\lambda)}{(-1+\mu)}=\frac{3}{1}$ or $\frac{\frac{1}{4}(4+\lambda)}{\frac{1}{4}(-1+\mu)}=\frac{3}{1}$	A1
	$\lambda-3\mu+7=0$ * Allow $0=\lambda-3\mu+7$ but nothing else.	A1*
		(4)
(b)	$\lambda=2 \Rightarrow \mu=3$; Resultant force $=(6\mathbf{i}+2\mathbf{j})$ (N)	M1
	$(6\mathbf{i}+2\mathbf{j})=4\mathbf{a}$ **OR** $\lvert(6\mathbf{i}+2\mathbf{j})\rvert=4a$	M1
	Use of $\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with $\mathbf{u}=\mathbf{0}$, their $\mathbf{a}$ and $t=4$: **Or** they may integrate their $\mathbf{a}$ twice with $\mathbf{u}=\mathbf{0}$ and put $t=4$: $\mathbf{r}=\frac{1}{2}\times\frac{(6\mathbf{i}+2\mathbf{j})}{4}4^2=(12\mathbf{i}+4\mathbf{j})$	DM1
	$\sqrt{12^2+4^2}$	M1
	ALTERNATIVE 1 for last two M marks: Use of $s=ut+\frac{1}{2}at^2$, with $u=0$, their a and $t=4$: DM1 $s=\frac{1}{2}\times\sqrt{1.5^2+0.5^2}\times4^2$ Use of Pythagoras to find mag of $\mathbf{a}$: $a=\sqrt{1.5^2+0.5^2}$ M1	
	ALTERNATIVE 2 for last two M marks: Use of $s=ut+\frac{1}{2}at^2$, with $u=0$, their a and $t=4$: DM1 $s=\frac{1}{2}\times\left(\frac{\sqrt{6^2+2^2}}{4}\right)\times4^2$ Use of Pythagoras to find $\lvert(6\mathbf{i}+2\mathbf{j})\rvert$: $=\sqrt{6^2+2^2}$ M1	
	$\sqrt{160}$, $2\sqrt{40}$, $4\sqrt{10}$ oe or 13 or better (m)	A1
		(5)

4. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P passes through the point A.

At time $t = 5$ s, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$

(a) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})$ m.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})$ m.

(b) Find the value of T.

(3)

(c) Find the value of c.

(3)

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Constant accn. with vectors

4(a)	$\mathbf{v}_B = (-16\mathbf{i} - 3\mathbf{j}) + 5(2.4\mathbf{i} + \mathbf{j})$	M1
	$\mathbf{v}_B = (-4\mathbf{i} + 2\mathbf{j})$	A1
	$\sqrt{(-4)^2 + 2^2}$	M1
	$\sqrt{20} = 2\sqrt{5}$, 4.5 or better ($\mathrm{m\ s^{-1}}$)	A1
		(4)
4(b)	<u>Using *A* as the initial position</u>: $\mathbf{r}_C = \mathbf{v}_A t + \frac{1}{2}\mathbf{a}t^2 + \mathbf{r}_A$ where *t* = *T* $(4\mathbf{i} + c\mathbf{j}) = (-16\mathbf{i} - 3\mathbf{j})T + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})T^2 + (44\mathbf{i} - 10\mathbf{j})$ **OR** $\begin{pmatrix} 4 \\ c \end{pmatrix} = \begin{pmatrix} -16 \\ -3 \end{pmatrix} T + \frac{1}{2}\begin{pmatrix} 2.4 \\ 1 \end{pmatrix} T^2 + \begin{pmatrix} 44 \\ -10 \end{pmatrix}$ Equating **i**-components, to give a **quadratic** equation in *T* only. Allow *t* instead of *T*.	M1
	$4 = -16T + \frac{1}{2} \times 2.4T^2 + 44$	A1
	(*T* =) 10	A1

4(c)	Equating **j**-components, with <u>their value of *T* or *t* substituted</u>, to give an equation, which must have a square term, in *c* only. **N.B.** Allow $\pm c$ in their equation. **(N.B.** Allow omission of -10 or their -12.5 for this M mark i.e. if using *A* as initial position $c = (-3 \times 10) + \frac{1}{2} \times 1 \times 10^2$ scores M1M0A0 **OR** if using *B* as initial position $c = (2 \times 5) + \frac{1}{2} \times 1 \times 5^2$ scores M1M0A0)	M1
	if using *A* as initial position $c = (-3 \times 10) + \frac{1}{2} \times 1 \times 10^2 + (-10)$ **N.B.** Allow $\pm c$ and/or $\pm(-10)$ in their equation **OR** if using *B* as initial position $c = (2 \times 5) + \frac{1}{2} \times 1 \times 5^2 + (-12.5)$ **N.B.** Allow $\pm c$ and/or $\pm(-12.5)$ in their equation	M1
	c = 10	A1
		(3)

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