edexcel

NAME:

Transition Exercises for A Level Mathematics

Please use these materials over the summer to practise the key skills you will need to be successful in the first term of A Level Mathematics.

Other recommended resources:

- **Starting Advanced Mathematics. The Essential Foundation** by Hugh Neill/Sarah Payne (ISBN: 9780521893565) published by Cambridge University Press
- **Head Start to Maths** (ISBN: 9781841469935) published by CGP Books
- **Bridging GCSE and A Level Maths** by Mark Rowland (ISBN: 9780007410231) published by Collins

Pearson

Expanding brackets and simplifying expressions

Practice

1 Expand.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$

c $-(3xy - 2y^2)$

> **Watch out!**
>
> When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '–'.

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

c $9(3s + 1) - 5(6s - 10)$ **d** $2(4x - 3) - (3x + 5)$

3 Expand.

a $3x(4x + 8)$ **b** $4k(5k^2 - 12)$

c $-2h(6h^2 + 11h - 5)$ **d** $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

c $4p(2p - 1) - 3p(5p - 2)$ **d** $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

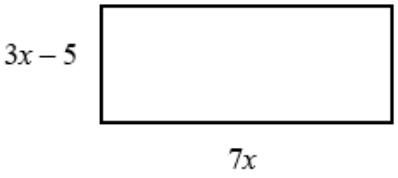
6 Expand and simplify.

a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.

Write down an expression, in terms of x, for the area of the rectangle.

Show that the area of the rectangle can be written as $21x^2 - 35x$



8 Expand and simplify.

a $(x + 4)(x + 5)$ **b** $(x + 7)(x + 3)$

c $(x + 7)(x - 2)$ **d** $(x + 5)(x - 5)$

e $(2x + 3)(x - 1)$ **f** $(3x - 2)(2x + 1)$

g $(5x - 3)(2x - 5)$ **h** $(3x - 2)(7 + 4x)$

i $(3x + 4y)(5y + 6x)$ **j** $(x + 5)^2$

k $(2x - 7)^2$ **l** $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a $\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$ **b** $\left(x+\frac{1}{x}\right)^2$

Surds and rationalising the denominator

Practice

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$

c $\sqrt{48}$ **d** $\sqrt{175}$

e $\sqrt{300}$ **f** $\sqrt{28}$

g $\sqrt{72}$ **h** $\sqrt{162}$

Hint

One of the two numbers you choose at the start must be a square number.

2 Simplify.

a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

Watch out!

Check you have chosen the highest square number at the start.

3 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$ **d** $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$ **f** $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$ **h** $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$ **b** $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\frac{1}{\sqrt{x}-\sqrt{y}}$

Rules of indices

Practice

1 Evaluate.

a 14^0 **b** 3^0 **c** 5^0 **d** x^0

2 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2} **b** 4^{-3} **c** 2^{-5} **d** 6^{-2}

5 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$ **b** $\frac{10x^5}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$ **d** $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

Watch out!

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

g $\frac{\left(2x^2\right)^3}{4x^0}$ **h** $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

6 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x.

a $\frac{1}{x}$ **b** $\frac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{1}{3x^4}$

d $\frac{2}{\sqrt{x}}$ **e** $\frac{4}{\sqrt[3]{x}}$ **f** 3

Extend

10 Write as sums of powers of x.

a $\frac{x^5+1}{x^2}$ **b** $x^2\left(x+\frac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\frac{1}{x^3}\right)$

Factorising expressions

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

> **Hint**
>
> Take the highest common factor outside the bracket.

2 Factorise

a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$

c $x^2 - 11x + 30$ **d** $x^2 - 5x - 24$

e $x^2 - 7x - 18$ **f** $x^2 + x - 20$

g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$

c $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

c $2x^2 + 7x + 3$ **d** $9x^2 - 15x + 4$

e $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2+4x}{x^2-x}$ **b** $\dfrac{x^2+3x}{x^2+2x-3}$

c $\dfrac{x^2-2x-8}{x^2-4x}$ **d** $\dfrac{x^2-5x}{x^2-25}$

e $\dfrac{x^2-x-12}{x^2-4x}$ **f** $\dfrac{2x^2+14x}{2x^2+4x-70}$

6 Simplify

a $\dfrac{9x^2-16}{3x^2+17x-28}$ **b** $\dfrac{2x^2-7x-15}{3x^2-17x+10}$

c $\dfrac{4-25x^2}{10x^2-11x-6}$ **d** $\dfrac{6x^2-x-1}{2x^2+7x-4}$

Extend

7 Simplify $\sqrt{x^2+10x+25}$

8 Simplify $\dfrac{(x+2)^2+3(x+2)^2}{x^2-4}$

Completing the square

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$

c $x^2 - 8x$ **d** $x^2 + 6x$

e $x^2 - 2x + 7$ **f** $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$

c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

3 Complete the square.

a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$

c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Solving quadratic equations

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

3 Solve by completing the square.

a $x^2 - 4x - 3 = 0$ **b** $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$ **d** $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$ **f** $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

> **Hint**
>
> Get all terms onto one side of the equation.

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

> **Hint**
>
> Get all terms onto one side of the equation.

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Sketching quadratic graphs

Practice

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Solving linear simultaneous equations

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

7 $y = x - 4$
$2x + 5y = 43$

8 $y = 2x - 3$
$5x - 3y = 11$

9 $2y = 4x + 5$
$9x + 5y = 22$

10 $2x = y - 2$
$8x - 5y = -11$

11 $3x + 4y = 8$
$2x - y = -13$

12 $3y = 4x - 7$
$2y = 3x - 4$

13 $3x = y - 1$
$2y - 2x = 3$

14 $3x + 2y + 1 = 0$
$4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x+y) = \frac{3(y-x)}{4}$.

Solving linear and quadratic simultaneous equations

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

Solving simultaneous equations graphically

Practice

1 Solve these pairs of simultaneous equations graphically.

a $y = 3x - 1$ and $y = x + 3$

b $y = x - 5$ and $y = 7 - 5x$

c $y = 3x + 4$ and $y = 2 - x$

2 Solve these pairs of simultaneous equations graphically.

a $x + y = 0$ and $y = 2x + 6$

b $4x + 2y = 3$ and $y = 3x - 1$

c $2x + y + 4 = 0$ and $2y = 3x - 1$

Hint

Rearrange the equation to make y the subject.

3 Solve these pairs of simultaneous equations graphically.

a $y = x - 1$ and $y = x^2 - 4x + 3$

b $y = 1 - 3x$ and $y = x^2 - 3x - 3$

c $y = 3 - x$ and $y = x^2 + 2x + 5$

4 Solve the simultaneous equations $x + y = 1$ and $x^2 + y^2 = 25$ graphically.

Extend

5 **a** Solve the simultaneous equations $2x + y = 3$ and $x^2 + y = 4$

i graphically

ii algebraically to 2 decimal places.

b Which method gives the more accurate solutions? Explain your answer.

Linear inequalities

Practice

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Quadratic inequalities

Practice

1 Find the set of values of x for which $(x + 7)(x – 4) \leq 0$

2 Find the set of values of x for which $x^2 – 4x – 12 \geq 0$

3 Find the set of values of x for which $2x^2 – 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x – 3 > 0$

5 Find the set of values of x for which $12 + x – x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x – 9) < –10$

8 $6x^2 \geq 15 + x$

Sketching cubic and reciprocal graphs

Practice

1 Here are six equations.

A $y = \frac{5}{x}$ **B** $y = x^2 + 3x – 10$ **C** $y = x^3 + 3x^2$

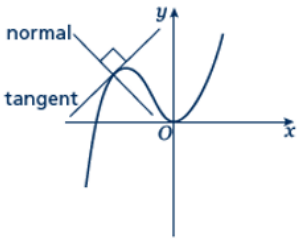
D $y = 1 – 3x^2 – x^3$ **E** $y = x^3 – 3x^2 – 1$ **F** $x + y = 5$

Hint

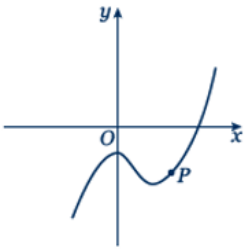
Find where each of the cubic equations cross the y-axis.

Here are six graphs.

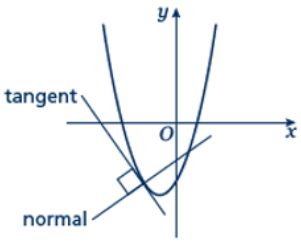
i



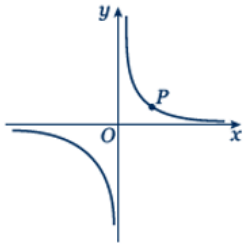
ii



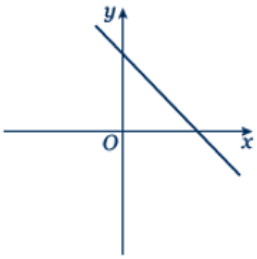
iii



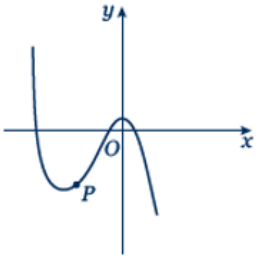
iv



v



vi



a Match each graph to its equation.

b Copy the graphs ii, iv and vi and draw the tangent and normal each at point *P*.

Sketch the following graphs

2 $y = 2x^3$

3 $y = x(x - 2)(x + 2)$

4 $y = (x + 1)(x + 4)(x - 3)$

5 $y = (x + 1)(x - 2)(1 - x)$

6 $y = (x - 3)^2(x + 1)$

7 $y = (x - 1)^2(x - 2)$

8 $y = \frac{3}{x}$

Hint: Look at the shape of $y = \frac{a}{x}$ in the second key point.

9 $y = -\frac{2}{x}$

Extend

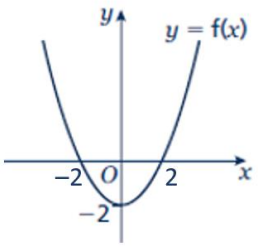
10 Sketch the graph of $y = \frac{1}{x+2}$

11 Sketch the graph of $y = \frac{1}{x-1}$

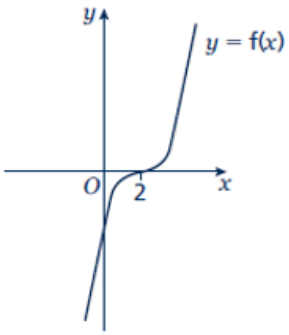
Translating graphs

Practice

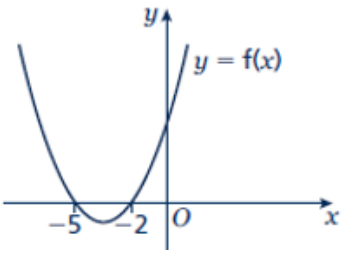
1 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x) + 4$ and $y = f(x + 2)$.



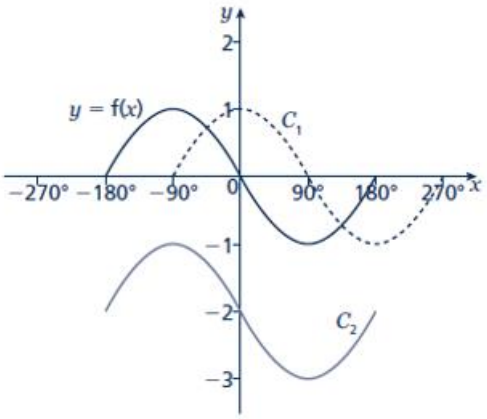
2 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x + 3)$ and $y = f(x) – 3$.



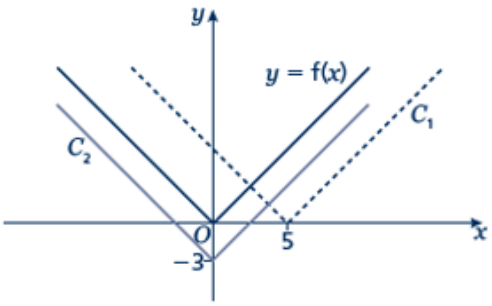
3 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch the graph of $y = f(x – 5)$.



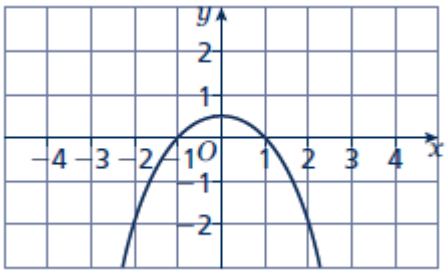
4 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2.
Write down the equations of the translated curves C_1 and C_2 in function form.



5 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2.
Write down the equations of the translated curves C_1 and C_2 in function form.



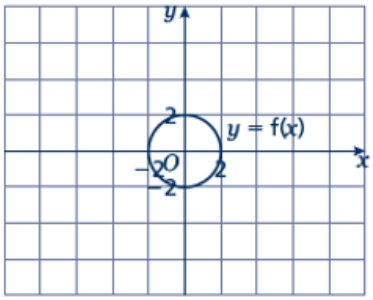
6 The graph shows the function $y = f(x)$.



a Sketch the graph of $y = f(x) + 2$

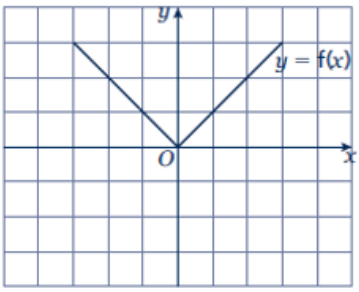
b Sketch the graph of $y = f(x + 2)$

7 The graph shows the function $y = f(x)$.

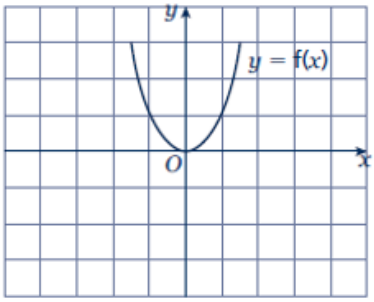


a Copy the graph and on the same axes sketch and label the graph of $y = 3f(x)$.

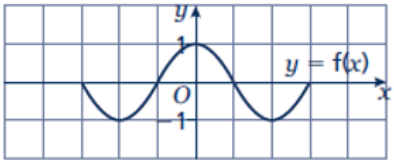
b Make another copy of the graph and on the same axes sketch and label the graph of $y = f(2x)$.



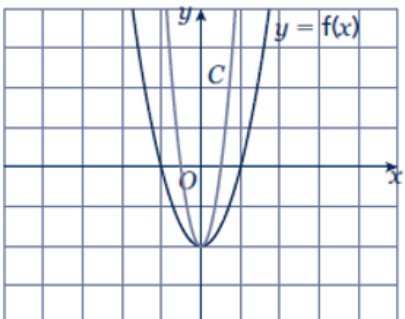
8 The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = -2f(x)$ and $y = f(3x)$.



9 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch and label the graphs of $y = -f(x)$ and $y = f\left(\frac{1}{2}x\right)$.

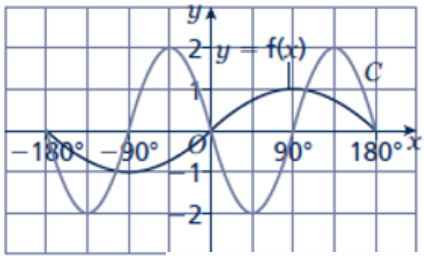


10 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch the graph of $y = -f(2x)$.



11 The graph shows the function $y = f(x)$ and a transformation, labelled C.
Write down the equation of the translated curve C in function form.

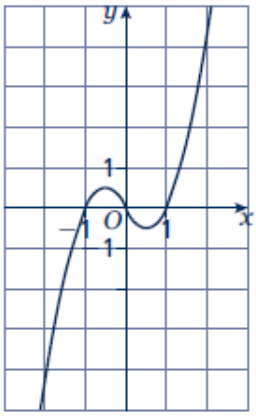
12 The graph shows the function $y = f(x)$ and a transformation labelled C.
Write down the equation of the translated curve C in function form.



13 The graph shows the function $y = f(x)$.

a Sketch the graph of $y = -f(x)$.

b Sketch the graph of $y = 2f(x)$.



Extend

14 **a** Sketch and label the graph of $y = f(x)$, where $f(x) = (x - 1)(x + 1)$.

b On the same axes, sketch and label the graphs of $y = f(x) - 2$ and $y = f(x + 2)$.

15 **a** Sketch and label the graph of $y = f(x)$, where $f(x) = -(x + 1)(x - 2)$.

b On the same axes, sketch and label the graph of $y = f\left(-\frac{1}{2}x\right)$.

Straight line graphs

Practice

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ **d** $x + y = 5$

e $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a gradient $-\frac{1}{2}$, y-intercept –7 **b** gradient 2, y-intercept 0

c gradient $\frac{2}{3}$, y-intercept 4 **d** gradient –1.2, y-intercept –2

4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

6 Write an equation for the line passing through each of the following pairs of points.

a (4, 5), (10, 17) **b** (0, 6), (–4, 8)

c (–1, –7), (5, 23) **d** (3, 10), (4, 7)

Extend

7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Parallel and Perpendicular Lines

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1 \quad (3, 2)$ **b** $y = 3 - 2x \quad (1, 3)$

c $2x + 4y + 3 = 0 \quad (6, -3)$ **d** $2y - 3x + 2 = 0 \quad (8, 20)$

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point $(-5, 3)$.

> **Hint**
> If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6 \quad (4, 0)$ **b** $y = -\frac{1}{3}x + \frac{1}{2} \quad (2, 13)$

c $x - 4y - 4 = 0 \quad (5, 15)$ **d** $5y + 2x - 5 = 0 \quad (6, 7)$

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a $(4, 3), \ (-2, -9)$ **b** $(0, 3), \ (-10, 8)$

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$, $y = 2x - 7$ **b** $y = 3x$, $2x + y - 3 = 0$ **c** $y = 4x - 3$, $4y + x = 2$

d $3x - y + 5 = 0$, $x + 3y = 1$ **e** $2x + 5y - 1 = 0$, $y = 2x + 7$ **f** $2x - y = 6$, $6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates $(-8, 3)$.

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

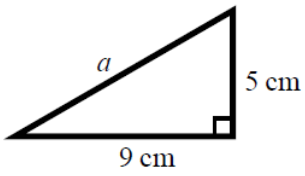
c Find an equation of $\mathbf{L_3}$

Pythagoras' Theorem

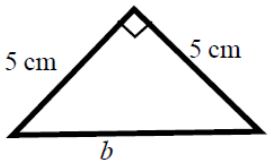
Practice

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

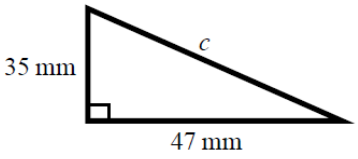
a



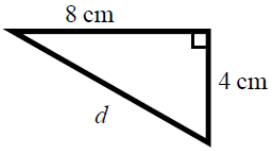
b



c

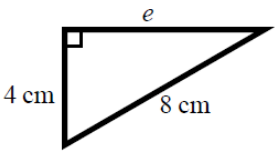


d

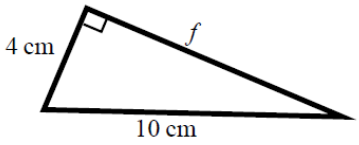


2 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

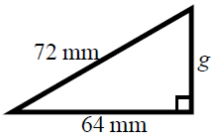
a



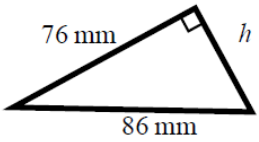
b



c

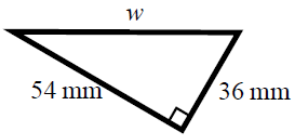


d

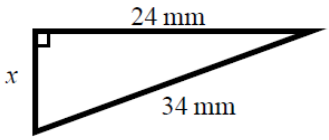


3 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

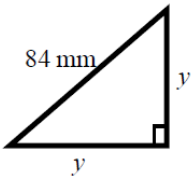
a



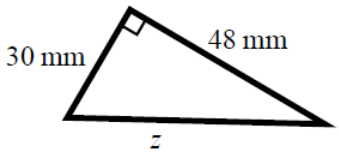
b



c



d



4 A rectangle has length 84 mm and width 45 mm.
Calculate the length of the diagonal of the rectangle.
Give your answer correct to 3 significant figures.

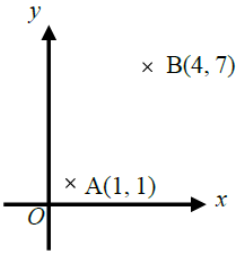
> **Hint**
>
> Draw a sketch of the rectangle.

Extend

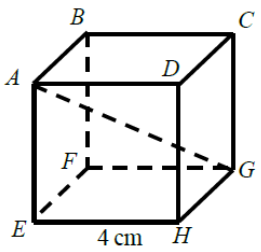
5 A yacht is 40 km due North of a lighthouse.
A rescue boat is 50 km due East of the same lighthouse.
Work out the distance between the yacht and the rescue boat.
Give your answer correct to 3 significant figures.

> **Hint**
>
> Draw a diagram using the information given in the question.

6 Points A and B are shown on the diagram.
Work out the length of the line AB.
Give your answer in surd form.



7 A cube has length 4 cm.
Work out the length of the diagonal *AG*.
Give your answer in surd form.



Proportion

Practice

Hint

Substitute the values given for P and h into the formula to calculate k.

1 Paul gets paid an hourly rate. The amount of pay (£P) is directly proportional to the number of hours (h) he works.
When he works 8 hours he is paid £56.
If Paul works for 11 hours, how much is he paid?

2 x is directly proportional to y.
$x = 35$ when $y = 5$.

a Find a formula for x in terms of y.

b Sketch the graph of the formula.

c Find x when $y = 13$.

d Find y when $x = 63$.

3 Q is directly proportional to the square of Z.
$Q = 48$ when $Z = 4$.

a Find a formula for Q in terms of Z.

b Sketch the graph of the formula.

c Find Q when $Z = 5$.

d Find Z when $Q = 300$.

4 y is directly proportional to the square of x.
$x = 2$ when $y = 10$.

a Find a formula for y in terms of x.

b Sketch the graph of the formula.

c Find x when $y = 90$.

5 B is directly proportional to the square root of C.
$C = 25$ when $B = 10$.

a Find B when $C = 64$.

b Find C when $B = 20$.

6 C is directly proportional to D.
$C = 100$ when $D = 150$.
Find C when $D = 450$.

7 y is directly proportional to x.
$x = 27$ when $y = 9$.
Find x when $y = 3.7$.

8 m is proportional to the cube of n.
$m = 54$ when $n = 3$.
Find n when $m = 250$.

Extend

9 s is inversely proportional to t.

a Given that $s = 2$ when $t = 2$, find a formula for s in terms of t.

b Sketch the graph of the formula.

c Find t when $s = 1$.

10 a is inversely proportional to b.
$a = 5$ when $b = 20$.

a Find a when $b = 50$.

b Find b when $a = 10$.

11 v is inversely proportional to w.
$w = 4$ when $v = 20$.

a Find a formula for v in terms of w.

b Sketch the graph of the formula.

c Find w when $v = 2$.

12 L is inversely proportional to W.
$L = 12$ when $W = 3$.
Find W when $L = 6$.

13 s is inversely proportional to t.
$s = 6$ when $t = 12$.

a Find s when $t = 3$.

b Find t when $s = 18$.

14 y is inversely proportional to x^2.
$y = 4$ when $x = 2$.
Find y when $x = 4$.

15 y is inversely proportional to the square root of x.
$x = 25$ when $y = 1$.
Find x when $y = 5$.

16 a is inversely proportional to b.
$a = 0.05$ when $b = 4$.

a Find a when $b = 2$.

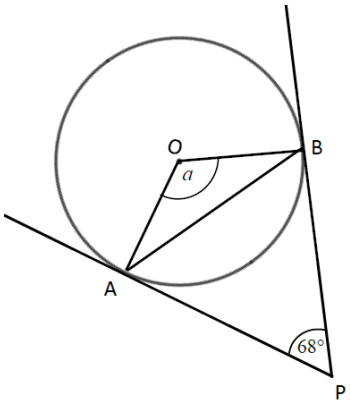
b Find b when $a = 2$.

Circle Theorems

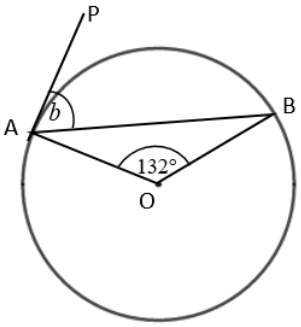
Practice

1 Work out the size of each angle marked with a letter.
Give reasons for your answers.

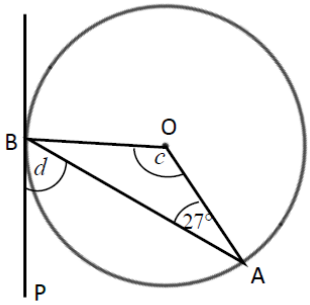
a



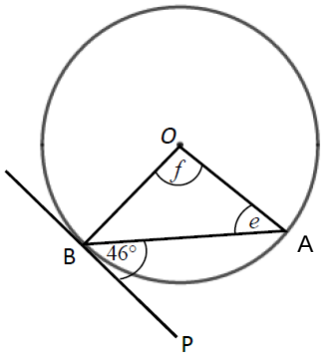
b



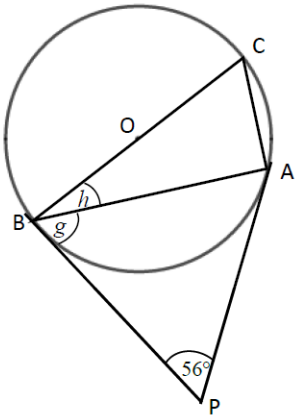
c



d

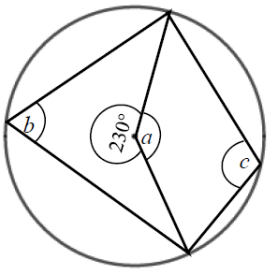


e

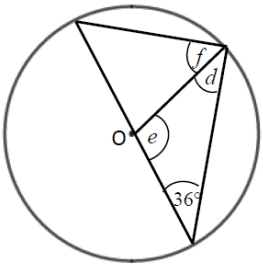


2 Work out the size of each angle marked with a letter.
Give reasons for your answers.

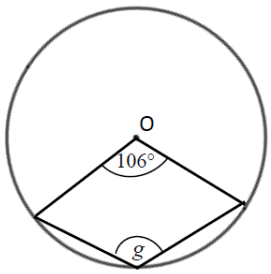
a



b



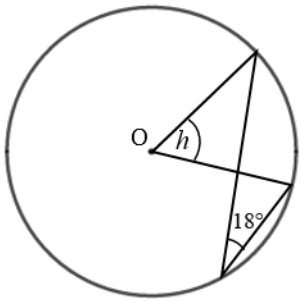
c



Hint

The reflex angle at point O and angle *g* are subtended by the same arc. So the reflex angle is twice the size of angle *g*.

d

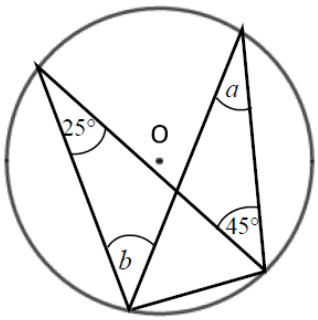


Hint

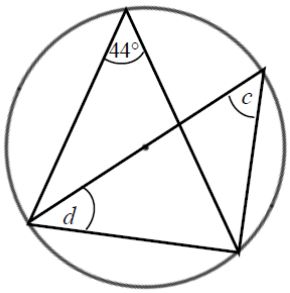
Angle 18° and angle *h* are subtended by the same arc.

3 Work out the size of each angle marked with a letter.
Give reasons for your answers.

a



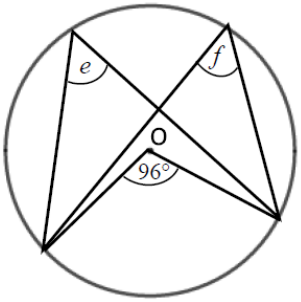
b



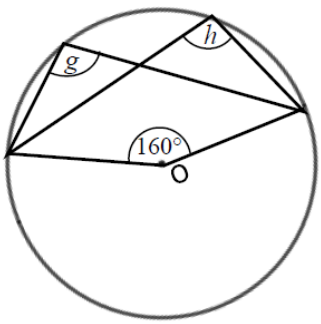
Hint

One of the angles is in a semicircle.

c

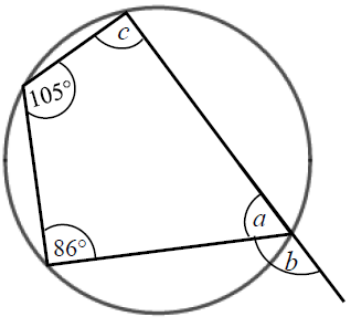


d



4 Work out the size of each angle marked with a letter.
Give reasons for your answers.

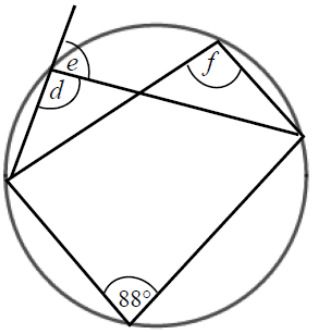
a



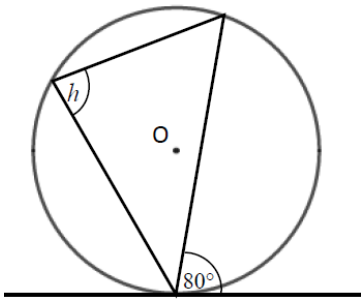
Hint

An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

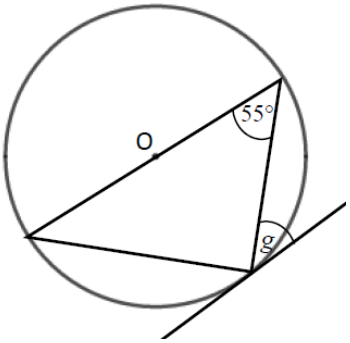
b



c



d



Hint

One of the angles is in a semicircle.

Extend

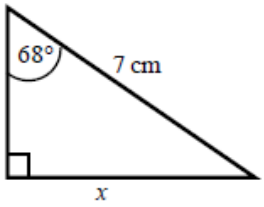
5 Prove the alternate segment theorem.

Trigonometry

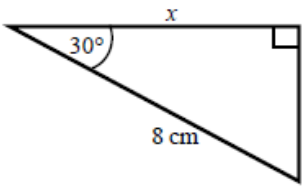
Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

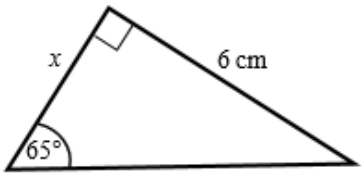
a



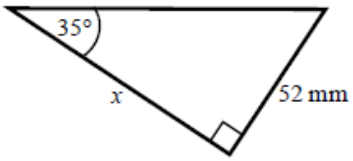
b



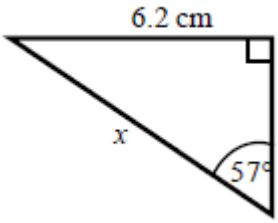
c



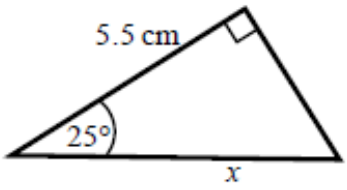
d



e

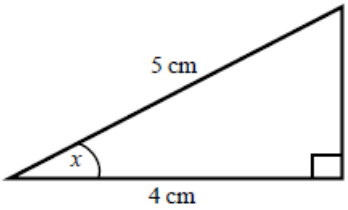


f

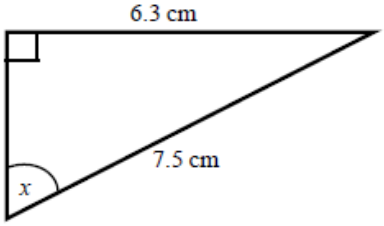


2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

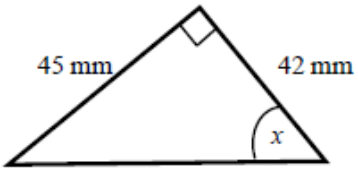
a



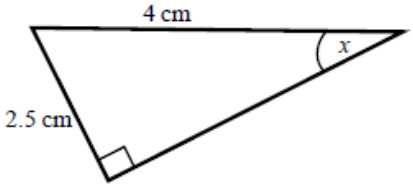
b



c

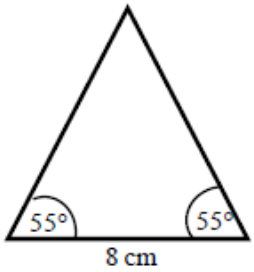


d

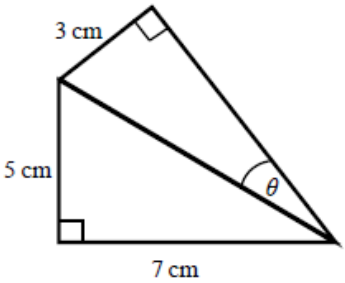


3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

> **Hint:**
>
> Split the triangle into two right-angled triangles.



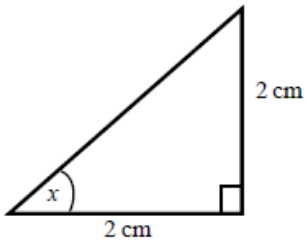
4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.



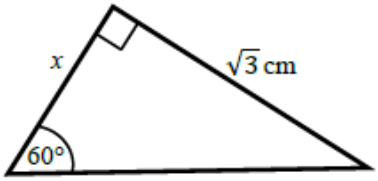
> **Hint:**
>
> First work out the length of the common side to both triangles, leaving your answer in surd form.

5 Find the exact value of x in each triangle.

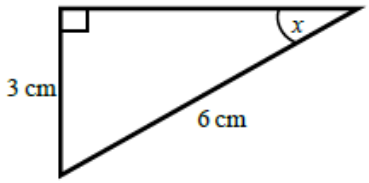
a



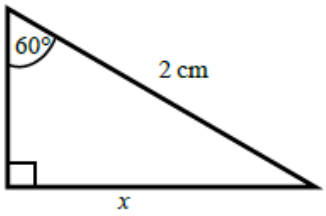
b



c

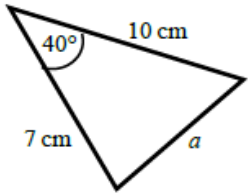


d

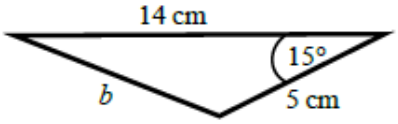


6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

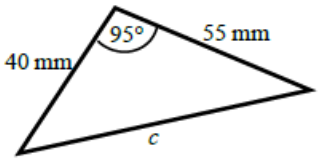
a



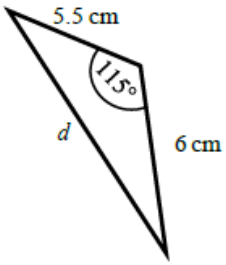
b



c

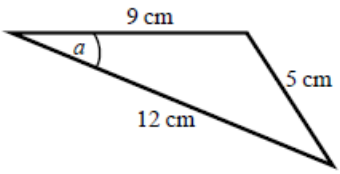


d

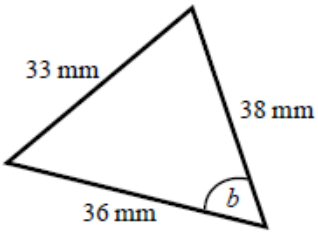


7 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

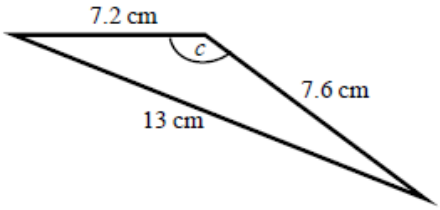
a



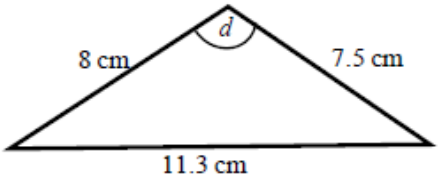
b



c

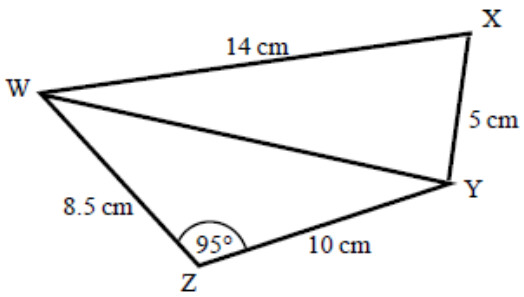


d



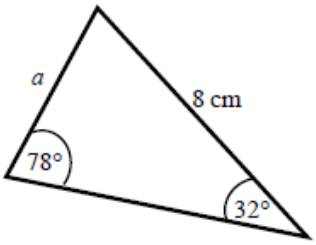
8 **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to
1 decimal place.

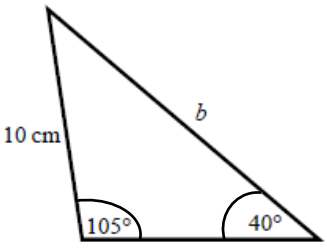


9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

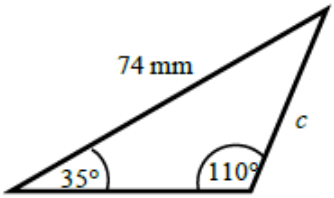
a



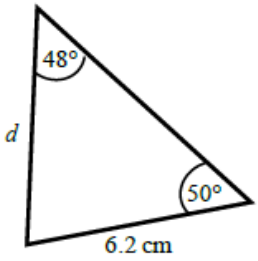
b



c

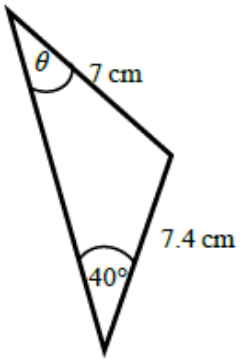


d

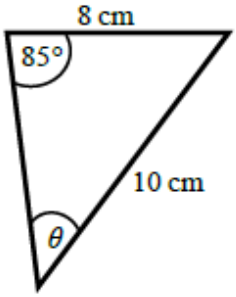


10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

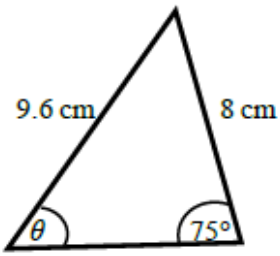
a



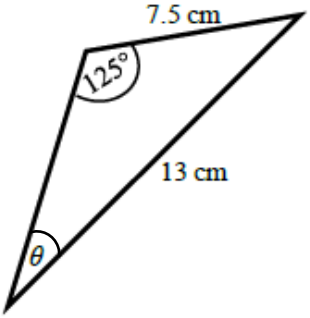
b



c

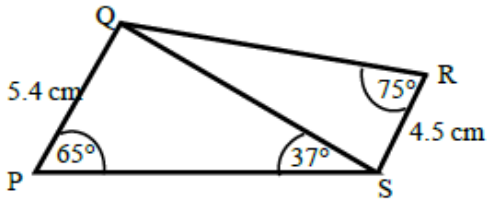


d



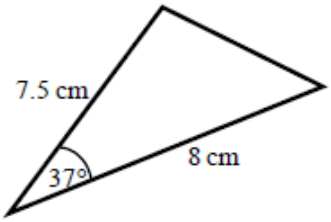
11 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.

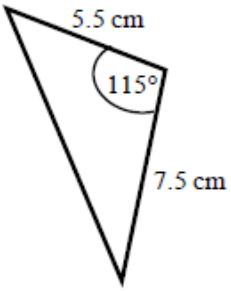


12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

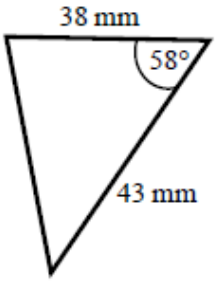
a



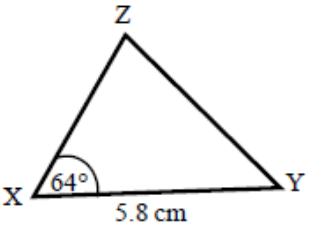
b



c



13 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.



Hint:

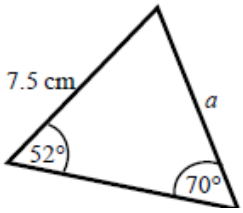
Rearrange the formula to make a side the subject.

Extend

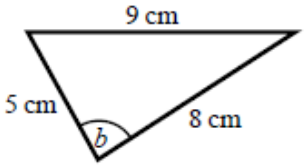
14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

> **Hint:**
>
> For each one, decide whether to use the cosine or sine rule.

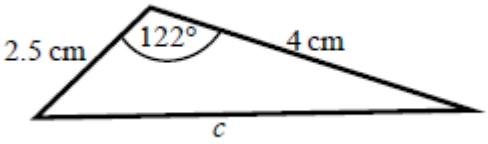
a



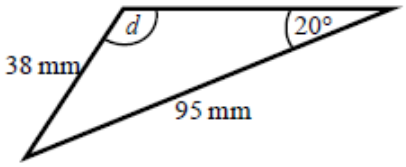
b



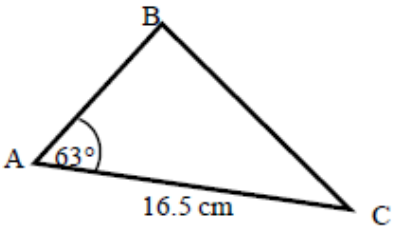
c



d



15 The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.



Rearranging Equations

Practice

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ $[d]$

2 $P = 2l + 2w$ $[w]$

3 $D = \frac{S}{T}$ $[T]$

4 $p = \frac{q-r}{t}$ $[t]$

5 $u = at - \frac{1}{2}t$ $[t]$

6 $V = ax + 4x$ $[x]$

7 $\frac{y-7x}{2} = \frac{7-2y}{3}$ $[y]$

8 $x = \frac{2a-1}{3-a}$ $[a]$

9 $x = \frac{b-c}{d}$ $[d]$

10 $h = \frac{7g-9}{2+g}$ $[g]$

11 $e(9 + x) = 2e + 1$ $[e]$

12 $y = \frac{2x+3}{4-x}$ $[x]$

13 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \frac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \frac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

a $\frac{xy}{z} = \frac{ab}{cd}$ **b** $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

15 Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make x the subject of the following equations.

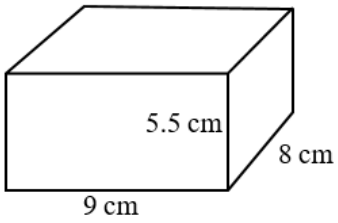
a $\frac{p}{q}(sx + t) = x - 1$ **b** $\frac{p}{q}(ax + 2y) = \frac{3p}{q^2}(x - y)$

Volume and Surface Area of 3D shapes

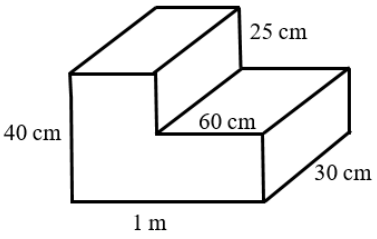
Practice

1 Work out the volume of each solid.
Leave your answers in terms of π where appropriate.

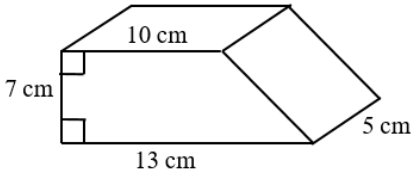
a



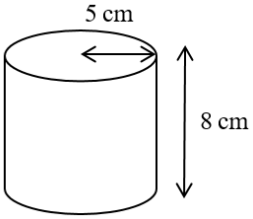
b



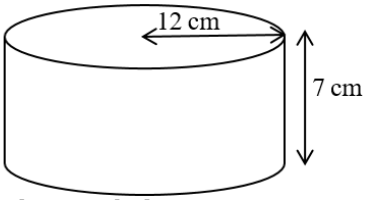
c



d



e

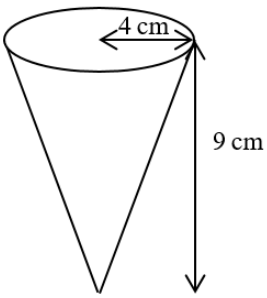


f a sphere with radius 7 cm

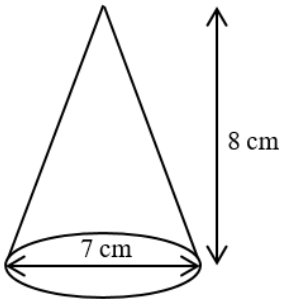
g a sphere with diameter 9 cm

h a hemisphere with radius 3 cm

i

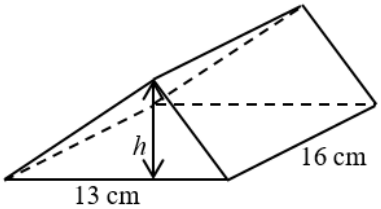


j



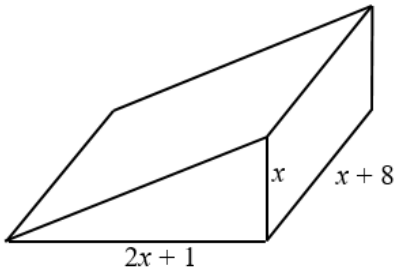
2 A cuboid has width 9.5 cm, height 8 cm and volume 1292 cm^3.
Work out its length.

3 The triangular prism has volume 1768 cm^3.
Work out its height.

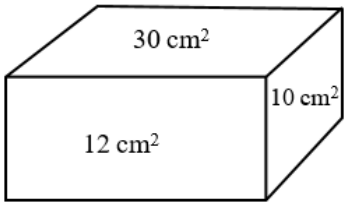


Extend

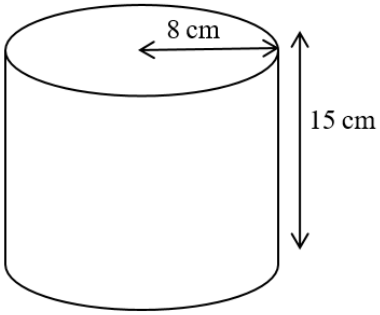
4 The diagram shows a solid triangular prism.
All the measurements are in centimetres.
The volume of the prism is V cm^3.
Find a formula for V in terms of x.
Give your answer in simplified form.



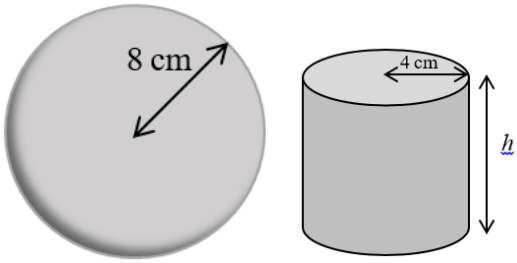
5 The diagram shows the area of each of three faces of a cuboid.
The length of each edge of the cuboid is a whole number of centimetres.
Work out the volume of the cuboid.



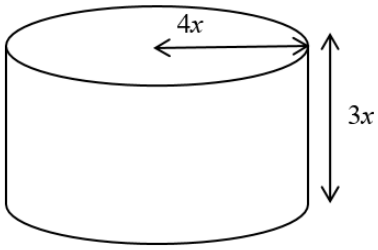
6 The diagram shows a large catering size tin of beans in the shape of a cylinder.
The tin has a radius of 8 cm and a height of 15 cm.
A company wants to make a new size of tin.
The new tin will have a radius of 6.7 cm.
It will have the same volume as the large tin.
Calculate the height of the new tin.
Give your answer correct to one decimal place.



7 The diagram shows a sphere and a solid cylinder.
The sphere has radius 8 cm.
The solid cylinder has a base radius of 4 cm and a height of h cm.
The total surface area of the cylinder is half the total surface area of the sphere.
Work out the ratio of the volume of the sphere to the volume of the cylinder.
Give your answer in its simplest form.



8 The diagram shows a solid metal cylinder.
The cylinder has base radius $4x$ and height $3x$.
The cylinder is melted down and made into a sphere of radius r.
Find an expression for r in terms of x.



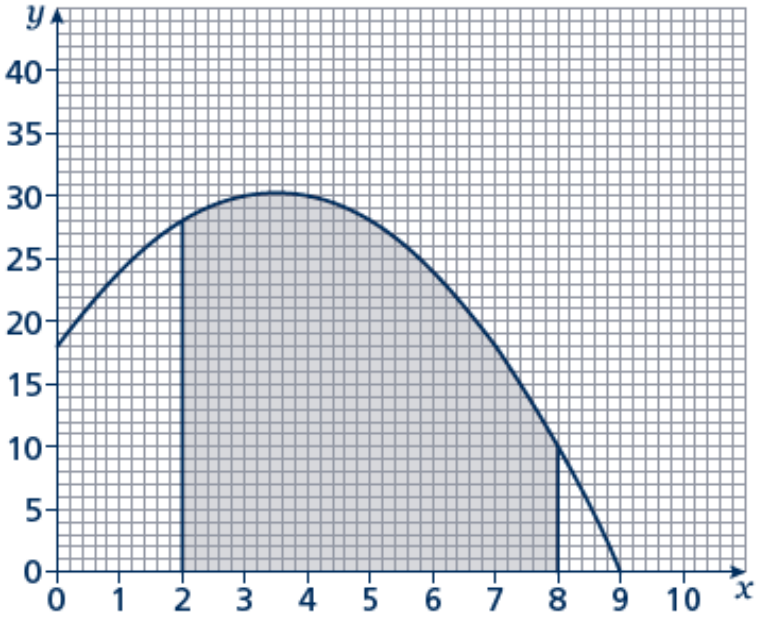
Area Under a Graph

Practice

1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

Hint:
For a full answer, remember to include 'units^2'.

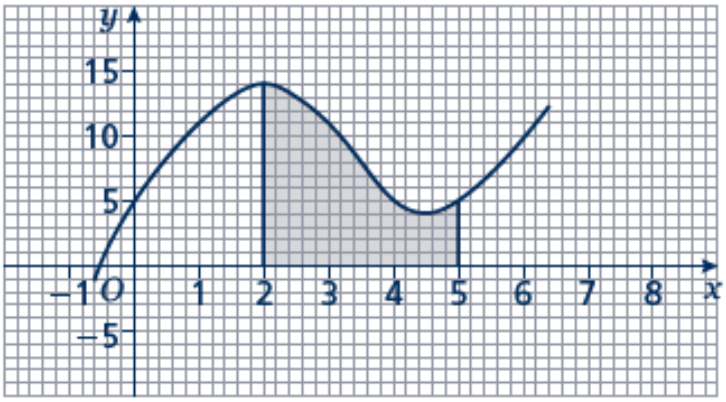
2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.



3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

4 Estimate the shaded area.

Use six strips of width $\frac{1}{2}$ unit.

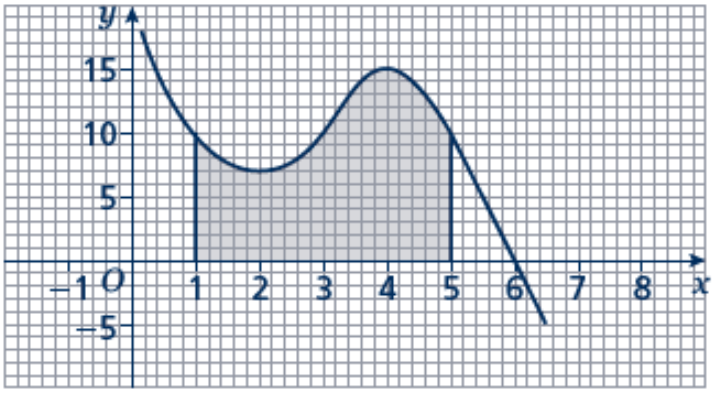


5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the x-axis from $x = -5$ to $x = 1$.

Use six strips of width 1 unit.

6 Estimate the shaded area.

Use four strips of equal width.

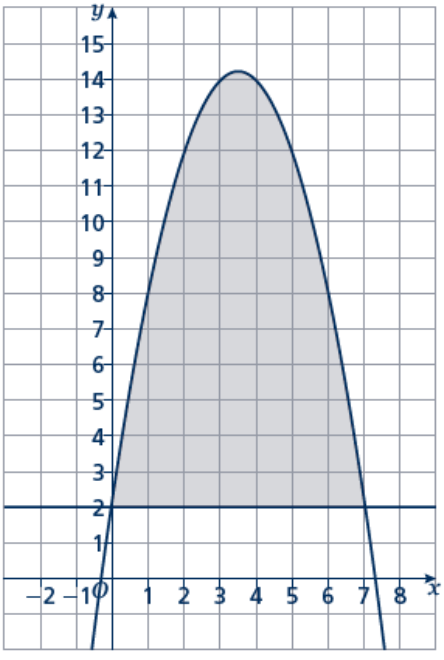


7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the x-axis from $x = 2$ to $x = 5$.

Use six strips of equal width.

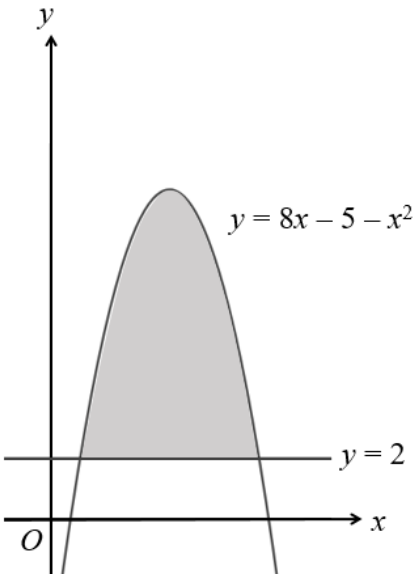
8 Estimate the shaded area.

Use seven strips of equal width.

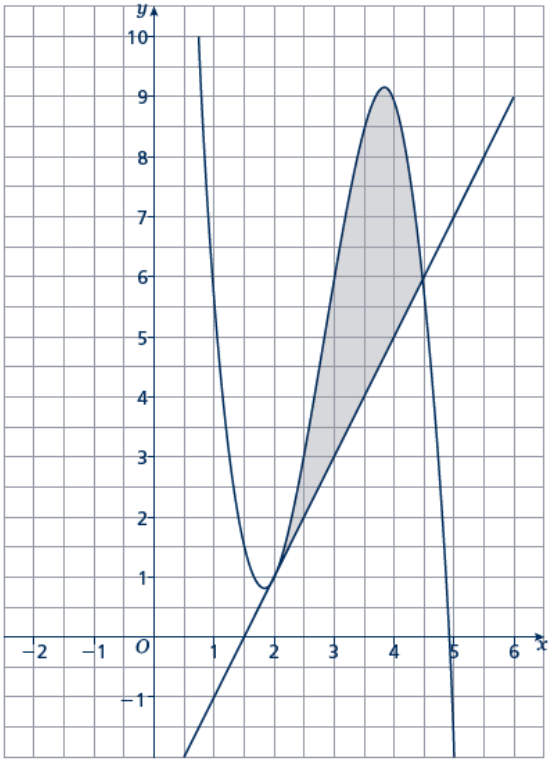


Extend

9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch.
Estimate the shaded area using six strips of equal width.



10 Estimate the shaded area using five strips of equal width.



Expanding brackets and simplifying expressions

Answers

1 **a** $6x - 3$ **b** $-10pq - 8q^2$
c $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$
b $40p - 16 - 12p - 27 = 28p - 43$
c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$
d $8x - 6 - 3x - 5 = 5x - 11$

3 **a** $12x^2 + 24x$ **b** $20k^3 - 48k$
c $10h - 12h^3 - 22h^2$ **d** $21s^2 - 21s^3 - 6s$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$
c $2p - 7p^2$ **d** $6b^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 $7x(3x - 5) = 21x^2 - 35x$

8 **a** $x^2 + 9x + 20$ **b** $x^2 + 10x + 21$
c $x^2 + 5x - 14$ **d** $x^2 - 25$
e $2x^2 + x - 3$ **f** $6x^2 - x - 2$
g $10x^2 - 31x + 15$ **h** $12x^2 + 13x - 14$
i $18x^2 + 39xy + 20y^2$ **j** $x^2 + 10x + 25$
k $4x^2 - 28x + 49$ **l** $16x^2 - 24xy + 9y^2$

9 $2x^2 - 2x + 25$

10 **a** $x^2 - 1 - \frac{2}{x^2}$ **b** $x^2 + 2 + \frac{1}{x^2}$

Surds and rationalising the denominator

Answers

1 **a** $3\sqrt{5}$ **b** $5\sqrt{5}$

c $4\sqrt{3}$ **d** $5\sqrt{7}$

e $10\sqrt{3}$ **f** $2\sqrt{7}$

g $6\sqrt{2}$ **h** $9\sqrt{2}$

2 **a** $15\sqrt{2}$ **b** $\sqrt{5}$

c $3\sqrt{2}$ **d** $\sqrt{3}$

e $6\sqrt{7}$ **f** $5\sqrt{3}$

3 **a** -1 **b** $9-\sqrt{3}$

c $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

4 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$

c $\frac{2\sqrt{7}}{7}$ **d** $\frac{\sqrt{2}}{2}$

e $\sqrt{2}$ **f** $\sqrt{5}$

g $\frac{\sqrt{3}}{3}$ **h** $\frac{1}{3}$

5 **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

6 $x-y$

7 **a** $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Rules of indices

Answers

1 **a** 1 **b** 1 **c** 1 **d** 1

2 **a** 7 **b** 4 **c** 5 **d** 2

3 **a** 125 **b** 32 **c** 343 **d** 8

4 **a** $\frac{1}{25}$ **b** $\frac{1}{64}$ **c** $\frac{1}{32}$ **d** $\frac{1}{36}$

5 **a** $\frac{3x^3}{2}$ **b** $5x^2$

c $3x$ **d** $\frac{y}{2x^2}$

e $y^{\frac{1}{2}}$ **f** c^{-3}

g $2x^6$ **h** x

6 **a** $\frac{1}{2}$ **b** $\frac{1}{9}$ **c** $\frac{8}{3}$

d $\frac{1}{4}$ **e** $\frac{4}{3}$ **f** $\frac{16}{9}$

7 **a** x^{-1} **b** x^{-7} **c** $x^{\frac{1}{4}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{3}}$ **f** $x^{-\frac{2}{3}}$

8 **a** $\frac{1}{x^3}$ **b** 1 **c** $\sqrt[5]{x}$

d $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt{x}}$ **f** $\frac{1}{\sqrt[4]{x^3}}$

9 **a** $5x^{\frac{1}{2}}$ **b** $2x^{-3}$ **c** $\frac{1}{3}x^{-4}$

d $2x^{-\frac{1}{2}}$ **e** $4x^{-\frac{1}{3}}$ **f** $3x^0$

10 **a** $x^3 + x^{-2}$ **b** $x^3 + x$ **c** $x^{-2} + x^{-7}$

Factorising expressions

Answers

1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$
c $5x^2y^2(5 - 2x + 3y)$

2 **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$
c $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$
e $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$
g $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$

3 **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$
c $2(3a - 10bc)(3a + 10bc)$

4 **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$
c $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$
e $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$

5 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$
c $\frac{x+2}{x}$ **d** $\frac{x}{x+5}$
e $\frac{x+3}{x}$ **f** $\frac{x}{x-5}$

6 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$
c $\frac{2-5x}{2x-3}$ **d** $\frac{3x+1}{x+4}$

7 $(x + 5)$

8 $\frac{4(x+2)}{x-2}$

Completing the square

Answers

1 **a** $(x + 2)^2 - 1$ **b** $(x - 5)^2 - 28$

c $(x - 4)^2 - 16$ **d** $(x + 3)^2 - 9$

e $(x - 1)^2 + 6$ **f** $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$

2 **a** $2(x - 2)^2 - 24$ **b** $4(x - 1)^2 - 20$

c $3(x + 2)^2 - 21$ **d** $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$

3 **a** $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$ **b** $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$

c $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$ **d** $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$

4 $(5x + 3)^2 + 3$

Solving quadratic equations

Answers

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$ **j** $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

3 **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ **b** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ **d** $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$ **f** $x = \frac{-3+\sqrt{89}}{10}$ or $x = \frac{-3-\sqrt{89}}{10}$

4 **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ **b** $x = \frac{-3+\sqrt{23}}{2}$ or $x = \frac{-3-\sqrt{23}}{2}$

c $x = \frac{5+\sqrt{13}}{2}$ or $x = \frac{5-\sqrt{13}}{2}$

5 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

6 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

7 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

8 **a** $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

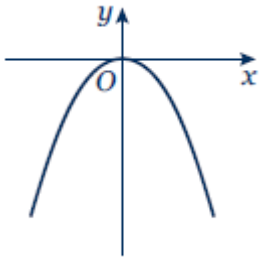
b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

c $x = -1\frac{2}{3}$ or $x = 2$

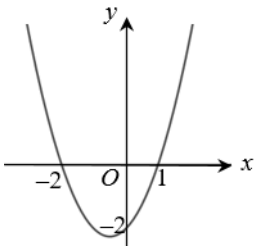
Sketching quadratic graphs

Answers

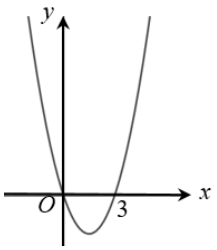
1



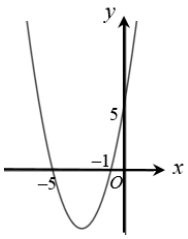
2 **a**



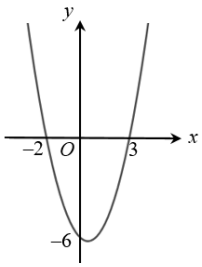
b



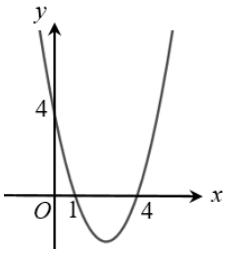
c



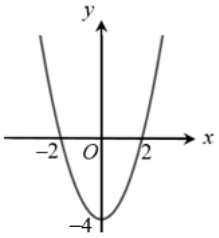
3 **a**



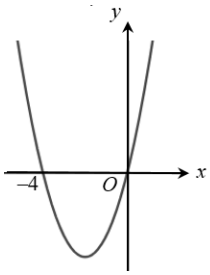
b



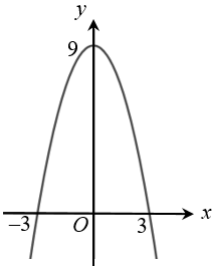
c



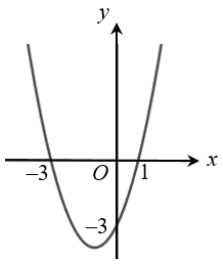
d



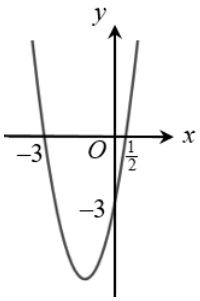
e



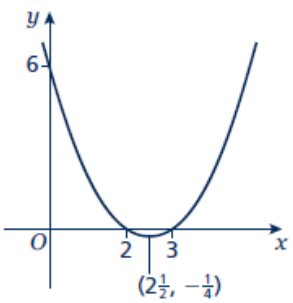
f



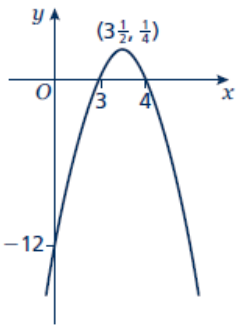
4



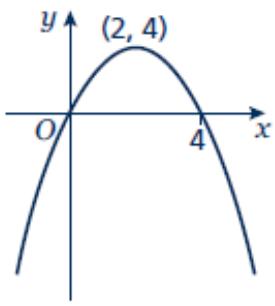
5 **a**



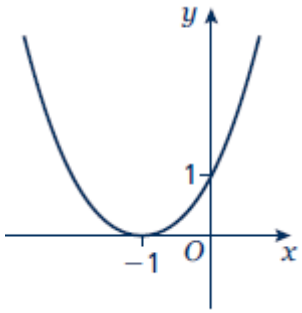
b



c



6



Line of symmetry at $x = -1$.

Solving linear simultaneous equations

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 9, y = 5$

8 $x = -2, y = -7$

9 $x = \frac{1}{2}, y = 3\frac{1}{2}$

10 $x = \frac{1}{2}, y = 3$

11 $x = -4, y = 5$

12 $x = -2, y = -5$

13 $x = \frac{1}{4}, y = 1\frac{3}{4}$

14 $x = -2, y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

Solving linear and quadratic simultaneous equations

Answers

1 $x = 1, y = 3$

$x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$

$x = 4, y = 2$

3 $x = 1, y = -2$

$x = 2, y = -1$

4 $x = 4, y = 1$

$x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$

$x = 2, y = 1$

6 $x = 7, y = 2$

$x = -1, y = -6$

7 $x = 0, y = 5$

$x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$

$x = 3, y = 5$

9 $x = -2, y = -4$

$x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$

$x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$

$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

Solving simultaneous equations graphically

Answers

1 **a** $x = 2, y = 5$
b $x = 2, y = -3$
c $x = -0.5, y = 2.5$

2 **a** $x = -2, y = 2$
b $x = 0.5, y = 0.5$
c $x = -1, y = -2$

3 **a** $x = 1, y = 0$ and $x = 4, y = 3$
b $x = -2, y = 7$ and $x = 2, y = -5$
c $x = -2, y = 5$ and $x = -1, y = 4$

4 $x = -3, y = 4$ and $x = 4, y = -3$

5 **a** **i** $x = 2.5, y = -2$ and $x = -0.5, y = 4$
ii $x = 2.41, y = -1.83$ and $x = -0.41, y = 3.83$
b Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.

Linear inequalities

Answers

1 **a** $x > 4$ **b** $x \leq 2$ **c** $x \leq -1$
d $x > -\frac{7}{2}$ **e** $x \geq 10$ **f** $x < -15$

2 **a** $x < -20$ **b** $x \leq 3.5$ **c** $x < 4$

3 **a** $x \leq -4$ **b** $-1 \leq x < 5$ **c** $x \leq 1$
d $x < -3$ **e** $x > 2$ **f** $x \leq -6$

4 **a** $t < \frac{5}{2}$ **b** $n \geq \frac{7}{5}$

5 **a** $x < -6$ **b** $x < \frac{3}{2}$

6 $x > 5$ (which also satisfies $x > 3$)

Quadratic inequalities

Answers

1 $-7 \leq x \leq 4$

2 $x \leq -2$ or $x \geq 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \leq x \leq 4$

6 $-3 \leq x \leq 2$

7 $2 < x < 2\frac{1}{2}$

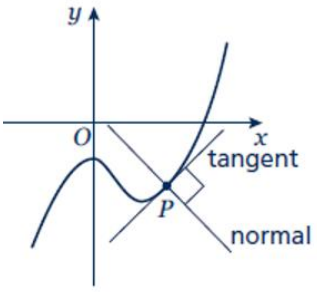
8 $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$

Sketching cubic and reciprocal graphs

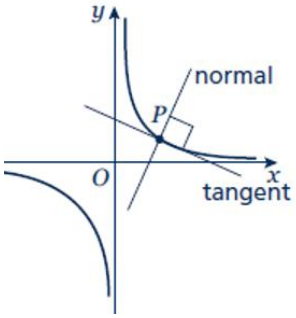
Answers

1 **a** i – C
ii – E
iii – B
iv – A
v – F
vi – D

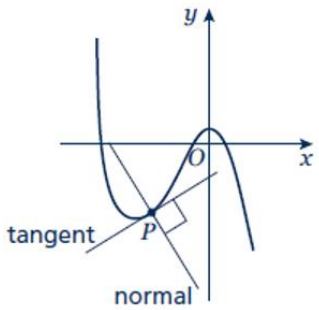
b **ii**



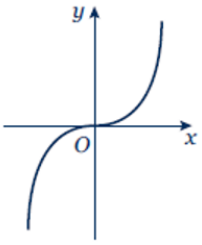
iv



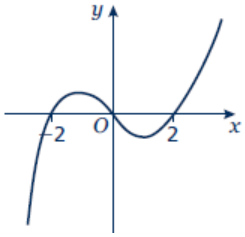
vi


y
O
x
tangent
P
normal

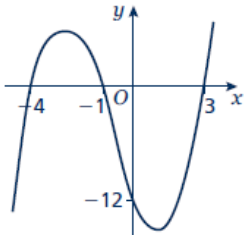
2


y
O
x

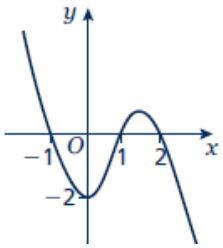
3


y
−2
O
2
x

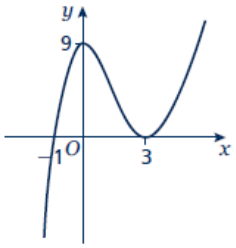
4


y
−4
−1
O
3
x
−12

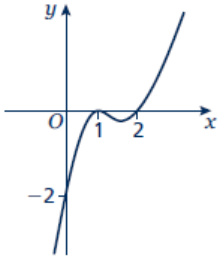
5


y
−1
O
1
2
x
−2

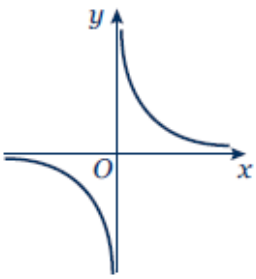
6


y
9
−1
O
3
x

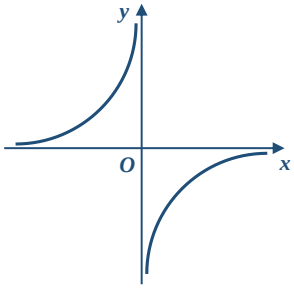
7


y
O
1
2
x
−2

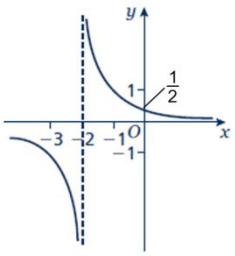
8



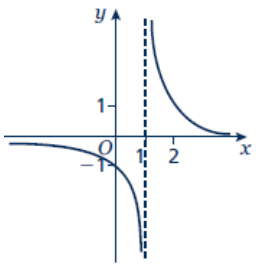
9



10



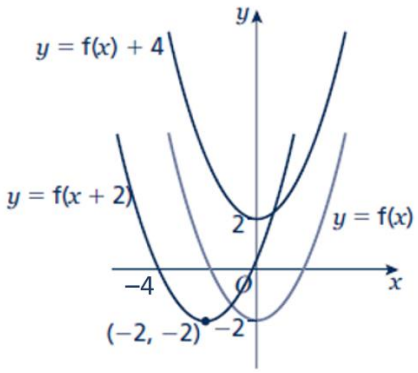
11



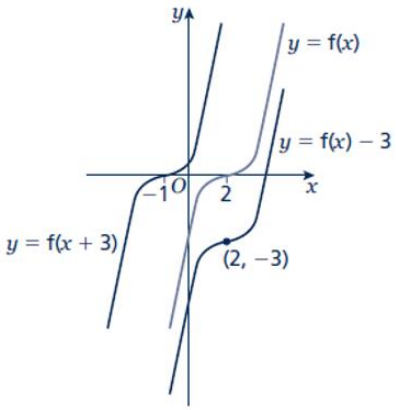
Translating graphs

Answers

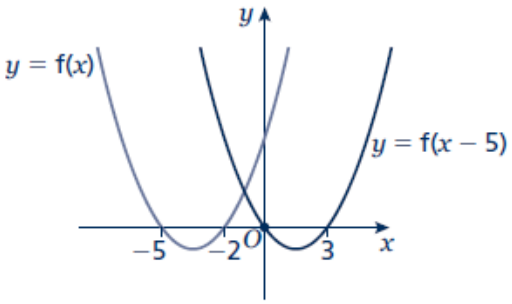
1



2



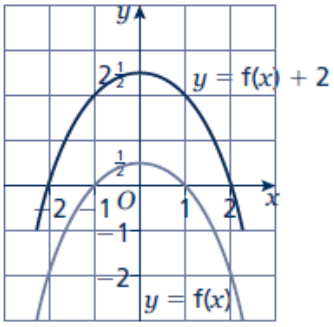
3



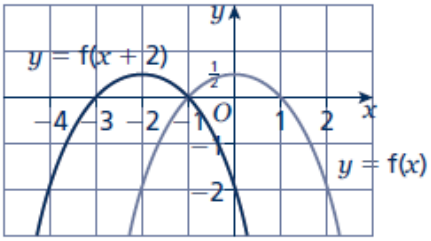
4 C_1: $y = f(x - 90°)$
C_2: $y = f(x) - 2$

5 C_1: $y = f(x - 5)$
C_2: $y = f(x) - 3$

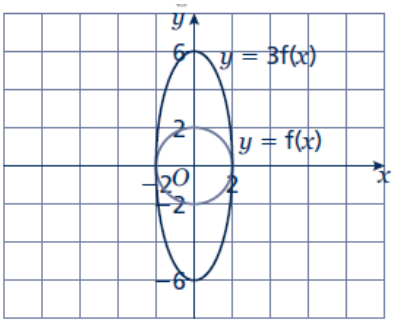
6 **a**



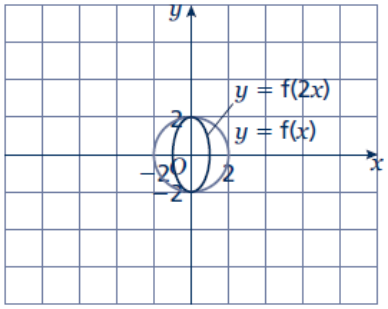
b



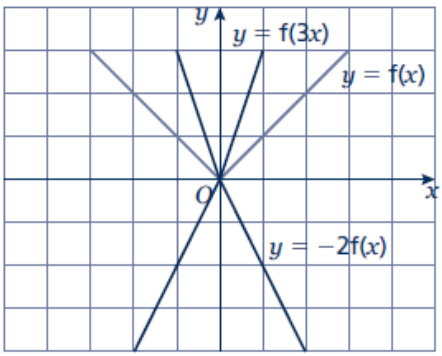
7 **a**



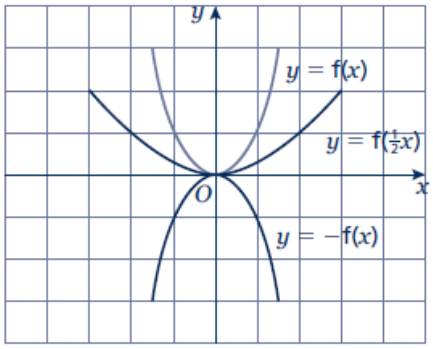
b



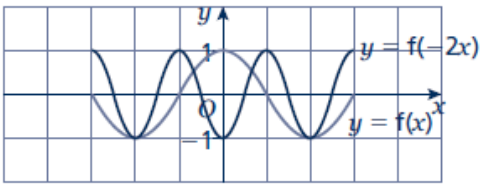
8



9



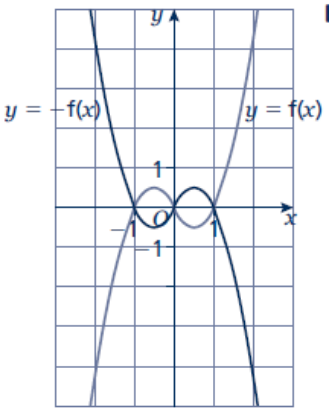
10



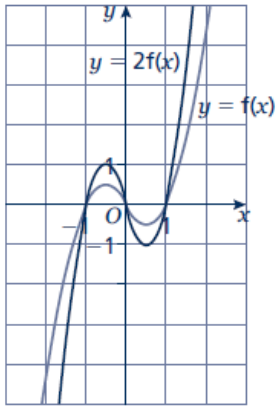
11 $y = f(2x)$

12 $y = -2f(2x)$ or $y = 2f(-2x)$

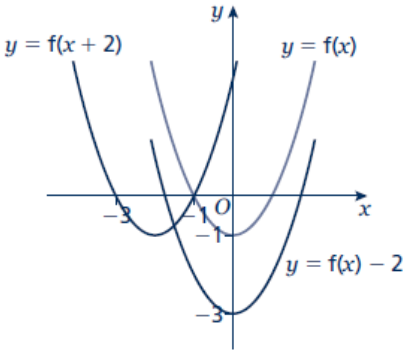
13 **a**



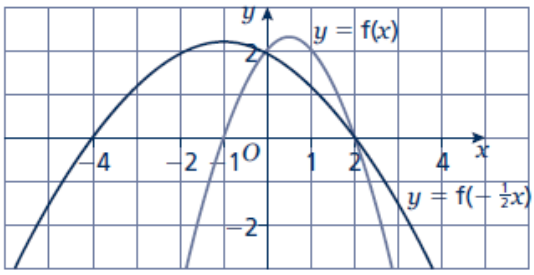
b



14



15



Straight Line Graphs

Answers

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

e $m = \frac{2}{3}$, $c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5$, $c = 4$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

c $y = 5x - 2$ **d** $y = -3x + 19$

7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the *y*-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

Parallel and perpendicular lines

Answers

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$

c $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\frac{1}{2}x + 2$ **b** $y = 3x + 7$

c $y = -4x + 35$ **d** $y = \frac{5}{2}x - 8$

4 **a** $y = -\frac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel **b** Neither **c** Perpendicular
d Perpendicular **e** Neither **f** Parallel

6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$

Pythagoras' Theorem

Answers

1 **a** 10.3 cm **b** 7.07 cm

c 58.6 mm **d** 8.94 cm

2 **a** $4\sqrt{3}$ cm **b** $2\sqrt{21}$ cm

c $8\sqrt{17}$ mm **d** $18\sqrt{5}$ mm

3 **a** $18\sqrt{13}$ mm **b** $2\sqrt{145}$ mm

c $42\sqrt{2}$ mm **d** $6\sqrt{89}$ mm

4 95.3 mm

5 64.0 km

6 $3\sqrt{5}$ units

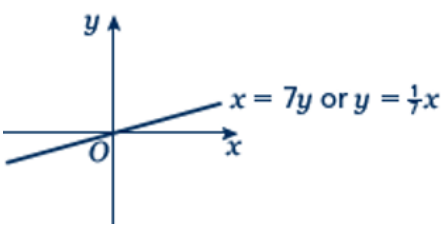
7 $4\sqrt{3}$ cm

Proportion

Answers

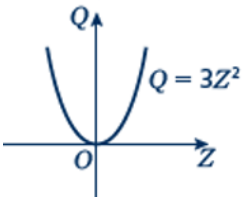
1 £77

2 **a** $x = 7y$ **b**



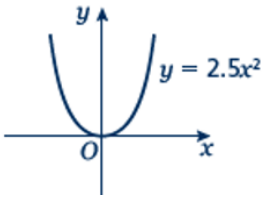
c 91 **d** 9

3 **a** $Q = 3Z^2$ **b**



c 75 **d** ±10

4 **a** $y = 2.5x^2$ **b**



c ±6

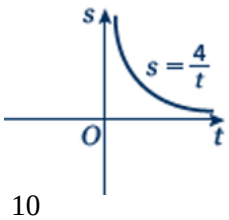
5 **a** 16 **b** 100

6 300

7 11.1

8 5

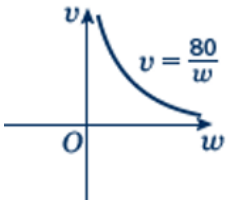
9 **a** $s = \frac{4}{t}$ **b**



c 4

10 **a** 2 **b** 10

11 **a** $v = \frac{80}{w}$ **b**



c 40

12 6

13 **a** 24 **b** 4

14 1

15 1

16 **a** 0.1 **b** 0.1

Circle Theorems

Answers

1 **a** $a = 112°$, angle OAP = angle OBP = 90° and angles in a quadrilateral total 360°.

b $b = 66°$, triangle OAB is isosceles, Angle OAP = 90° as AP is tangent to the circle.

c $c = 126°$, triangle OAB is isosceles.
$d = 63°$, Angle OBP = 90° as BP is tangent to the circle.

d $e = 44°$, the triangle is isosceles, so angles e and angle OBA are equal. The angle OBP = 90° as BP is tangent to the circle.
$f = 92°$, the triangle is isosceles.

e $g = 62°$, triangle ABP is isosceles as AP and BP are both tangents to the circle.
$h = 28°$, the angle OBP = 90°.

2 **a** $a = 130°$, angles in a full turn total 360°.
$b = 65°$, the angle at the centre of a circle is twice the angle at the circumference.
$c = 115°$, opposite angles in a cyclic quadrilateral total 180°.

b $d = 36°$, isosceles triangle.
$e = 108°$, angles in a triangle total 180°.
$f = 54°$, angle in a semicircle is 90°.

c $g = 127°$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.

d $h = 36°$, the angle at the centre of a circle is twice the angle at the circumference.

3 **a** $a = 25°$, angles in the same segment are equal.
$b = 45°$, angles in the same segment are equal.

b $c = 44°$, angles in the same segment are equal.
$d = 46°$, the angle in a semicircle is 90° and the angles in a triangle total 180°.

c $e = 48°$, the angle at the centre of a circle is twice the angle at the circumference.
$f = 48°$, angles in the same segment are equal.

d $g = 100°$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.
$h = 100°$, angles in the same segment are equal.

4 **a** $a = 75°$, opposite angles in a cyclic quadrilateral total 180°.
$b = 105°$, angles on a straight line total 180°.
$c = 94°$, opposite angles in a cyclic quadrilateral total 180°.

b $d = 92°$, opposite angles in a cyclic quadrilateral total 180°.
$e = 88°$, angles on a straight line total 180°.
$f = 92°$, angles in the same segment are equal.

c $h = 80°$, alternate segment theorem.

d $g = 35°$, alternate segment theorem and the angle in a semicircle is 90°.

5 Angle BAT = x.

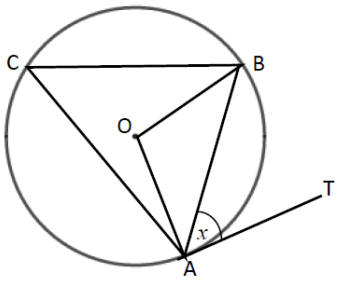
Angle OAB = $90° - x$ because the angle between the tangent and the radius is 90°.

OA = OB because radii are equal.

Angle OAB = angle OBA because the base of isosceles triangles are equal.

Angle AOB = $180° - (90° - x) - (90° - x) = 2x$ because angles in a triangle total 180°.

Angle ACB = $2x \div 2 = x$ because the angle at the centre is twice the angle at the circumference.



Trigonometry

Answers

1	**a**	6.49 cm	**b**	6.93 cm	**c**	2.80 cm		
	d	74.3 mm	**e**	7.39 cm	**f**	6.07 cm		
2	**a**	36.9°	**b**	57.1°	**c**	47.0°	**d**	38.7°
3	5.71 cm							
4	20.4°							
5	**a**	45°	**b**	1 cm	**c**	30°	**d**	$\sqrt{3}$ cm
6	**a**	6.46 cm	**b**	9.26 cm	**c**	70.8 mm	**d**	9.70 cm
7	**a**	22.2°	**b**	52.9°	**c**	122.9°	**d**	93.6°
8	**a**	13.7 cm	**b**	76.0°				
9	**a**	4.33 cm	**b**	15.0 cm	**c**	45.2 mm	**d**	6.39 cm
10	**a**	42.8°	**b**	52.8°	**c**	53.6°	**d**	28.2°
11	**a**	8.13 cm	**b**	32.3°				

12 **a** 18.1 cm^2 **b** 18.7 cm^2 **c** 693 mm^2

13 5.10 cm

14 **a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°

15 15.3 cm

Rearranging Equations

Answers

1 $d = \frac{C}{\pi}$ **2** $w = \frac{P-2l}{2}$ **3** $T = \frac{S}{D}$

4 $t = \frac{q-r}{p}$ **5** $t = \frac{2u}{2a-1}$ **6** $x = \frac{V}{a+4}$

7 $y = 2 + 3x$ **8** $a = \frac{3x+1}{x+2}$ **9** $d = \frac{b-c}{x}$

10 $g = \frac{2h+9}{7-h}$ **11** $e = \frac{1}{x+7}$ **12** $x = \frac{4y-3}{2+y}$

13 **a** $r = \sqrt{\frac{A}{\pi}}$ **b** $r = \sqrt[3]{\frac{3V}{4\pi}}$

c $r = \frac{P}{\pi+2}$ **d** $r = \sqrt{\frac{3V}{2\pi h}}$

14 **a** $x = \frac{abz}{cdy}$ **b** $x = \frac{3dz}{4\pi cpy^2}$

15 $\sin B = \frac{b\sin A}{a}$

16 $\cos B = \frac{a^2+c^2-b^2}{2ac}$

17 **a** $x = \frac{q+pt}{q-ps}$ **b** $x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$

Volume and Surface Area of 3D shapes

Answers

1 **a** $V = 396\ \text{cm}^3$ **b** $V = 75\,000\ \text{cm}^3$

c $V = 402.5\ \text{cm}^3$ **d** $V = 200\pi\ \text{cm}^3$

e $V = 1008\pi\ \text{cm}^3$ **f** $V = \frac{1372}{3}\pi\ \text{cm}^3$

g $V = 121.5\pi\ \text{cm}^3$ **h** $V = 18\pi\ \text{cm}^3$

i $V = 48\pi\ \text{cm}^3$ **j** $V = \frac{98}{3}\pi\ \text{cm}^3$

2 17 cm

3 17 cm

4 $V = x^3 + \frac{17}{2}x^2 + 4x$

5 $60\ \text{cm}^3$

6 21.4 cm

7 32 : 9

8 $r = \sqrt[3]{36x}$

Area Under a Graph

Answers

1 34 units^2

2 149 units^2

3 14 units^2

4 $25\frac{1}{4}$ units^2

5 35 units^2

6 42 units^2

7 $26\frac{7}{8}$ units^2

8 56 units^2

9 35 units^2

10 $6\frac{1}{4}$ units^2